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# Mathematical Description of the Process of Cutting a Melon Fruit With A Blade

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**Annotation.** The article is devoted to the mathematical description of the process of cutting elastic-viscous materials on the example of a melon fruit. A method for deriving theoretically calculated equations for determining the critical cutting force and the destructive contact stress is given. A schematic diagram of an experimental laboratory installation for determining the elastic modulus and Poisson's ratio of plant raw materials and the principle of its operation are described.

**Keywords:** elastic-viscous material, cutting, process, force, contact stress, blade, deformation modulus, relative compression, coefficient of friction, installation, pressing, melon.

**INTRODUCTION.**

When processing any material, it is necessary to have information about their physical, mechanical and technological properties, that is, about the properties that promote or counteract this type of mechanical processing, especially when cutting elastic-viscous products with a blade, such as meat or vegetable products. When processing these materials, the interaction of the blade with the base of the material is characterized by complex physical phenomena that are difficult to describe analytically, in contrast to Hooke's law. Therefore, only when combining theoretical calculations with full-scale experimental studies, it is possible to understand the true physical essence of the process. Taking into account some assumptions and assumptions, it is possible to determine theoretically the main factors affecting the process under study, and subsequent experimental studies show the adequacy of the accepted judgments [1, 2].

Elastic-viscous materials can, with some assumptions, include the model of melon pulp, which consists of a solid skeleton (cellulose fiber) and a semi-liquid substance that fills the gaps between the solid elements. Being deformed by the action of the knife blade, the fibers will press on the liquid medium, forcing it to move to less stressed areas. In accordance with the laws of hydrodynamics, the resistance of the medium during such a movement depends on the speed of its movement, that is, it states the fact that in viscous bodies, the deformation is a function of the load and the time of its action. Generically, the model of plant material, from the point of view of rheology, can be considered as a model obeying the Hooke-Newton law [3-5].

**MATERIALS AND METHODS OF RESEARCH.**

It is known that the separation of an elastic-viscous material into parts under the influence of the blade of a cutting tool occurs when a destructive contact stress occurs $\sigma_p$, which is determined by the value of the critical force applied to the blade $P_{кр}$.

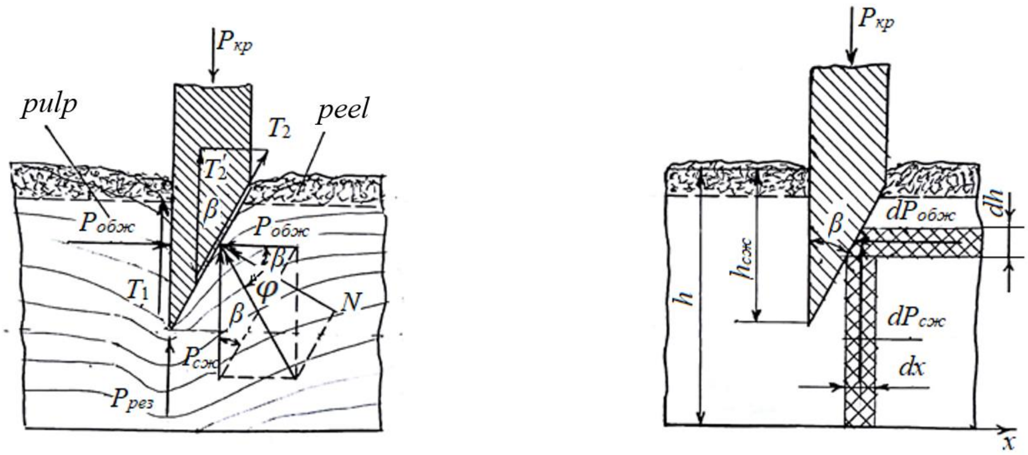
Let us consider the theoretical background of the process of cutting elastic-viscous materials, for example, the mechanical processing of the melon fruit by turning, that is, the removal of the peel by cutting with a blade. If we consider the process of cutting in time, then first there is a process of compression of the surface layers of the material (in our case, the peel and subcortical layer of the melon), and then the introduction of the blade into the pulp of the fruit. This assumption is illustrated in Figures 1, *a* and *b* [6-8].

When the blade is inserted into the flesh of a melon, the following forces act on it: $P_{рез}$ – blade edge resistance; $P_{обж}$ – resistance of the compression force in the horizontal plane; $P_{сж}$ – resistance of the compression layer by the chamfer of the blade along the course of the blade.

Geometrically decomposing into the components of the force $P_{обж}$ and $P_{сж}$, we determine the force acting on the chamfer $N$

$$N = P_{обж} \cos\beta + P_{сж} \sin\beta \,. \qquad (1)$$

When the blade moves on its vertical and oblique chamfers, friction forces $T_1$ and $T_2$ occur




a) b)

**Fig. 1. Schematic diagram of the force interaction of the blade with the melon fruit**

$$T_1 = f\ P_{обж} \qquad (2)$$

and

$$T_2 = f\ N\,, \qquad (3)$$

where $f = tg\ \varphi$, here $\varphi$ – is the angle of friction.

Then the force N can be expressed in terms of the angle of friction

$$N = \sqrt{P_{обж}^2 + P_{сж}^2} \cdot \cos\varphi\,, \qquad (4)$$

schematically, the force $T_1$ is directed upwards, and $T_2$ is directed at the angle $\beta$ of the chamfer inclination.

The vertical component of the force $T_2$ is equal to

$$T_2' = T_2 \cos\beta. \qquad (5)$$

Substituting the value of $N$ from (4) to (3) we get

$$T_2 = f\sqrt{P_{обж}^2 + P_{сж}^2} \cdot \cos\beta\,, \qquad (6)$$

well, taking into account (1), (3) and (5)

$$T_2' = f\,(\frac{1}{2} P_{сж} \sin 2\beta + P_{обж} \cos^2\beta). \qquad (7)$$

The cutting start condition is met when the sum of all forces acting in the vertical plane is less than the critical force $P_{кр}$ applied to the blade, i.e.

$$P_{кр} \geq P_{рез} + P_{сж} + T_1 + T_2' \ . \quad (8)$$

The force $P_{рез}$, can be defined as the product of the area of the cutting edge of the blade $F_{кр}$ by the destructive contact stress $\sigma_p$

$$P_{рез} = \sigma_p F_{кр} = \delta \Delta l \, \sigma_p, \quad (9)$$

where $\delta$ – the thickness of the blade; $\Delta l$ – the length of the blade.

To reveal the value of the $P_{сж}$ and $P_{рез}$ included in expression (8), we consider the forces acting on the elementary volume of the pulp from the side of the oblique chamfer of the blade (Fig.1, *b*). In this case, the relative compression $\varepsilon_{сж}$ of the selected elementary volume from the side of the oblique chamfer is equal to

$$\varepsilon_1 = \varepsilon_{сж} = \frac{h_{сж(x)}}{h}. \quad (10)$$

To simplify the problem, we express the relative compression in terms of the stress $\sigma$ and the strain modulus $E$, i.e. $\varepsilon_{сж} = \sigma / E$.

It would be logical to judge that $\varepsilon_{сж}$ is proportionally dependent on however $\sigma$, as shown by preliminary experiments, with an increase in $\varepsilon_{сж}$, the voltage $\sigma = P_{сж} / F_x$ growth lags behind the rate of growth of the $P_{сж}$ force and changes in a curvilinear relationship

$$F_x = \Delta l \, h_{сж} \, tg \, \beta. \quad (11)$$

In this case, taking the voltage as a function of the ratio $P_{сж}$ to the initial area, we obtain a power dependence

$$\varepsilon_{сж} \cdot E = \sigma^n, \quad (12)$$

where is the exponent of $n=1$.

Then the elementary compressive force $dP_{сж}$, acting on the area $dF$, with a length equal to one and a width $dx$, is represented as

$$dP_{сж} = E \varepsilon_{сж} \, dh_{сж} \, tg \beta \quad (13)$$

or substituting (10) in (13), we get

$$dP_{сж} = E \frac{h_{сж(x)}}{h} dh_{сж} \ tg\beta. \quad (14)$$

Integrating the expression (14) gives

$$P_{сж} = \frac{E}{2h} h_{сж}^2 \, tg\beta, \quad (15)$$

which shows that the required force of the $P_{сж}$ to compress the layer by the chamfer of the blade is in a quadratic dependence on the value of the $h_{сж}$ and graphically represents a parabola.

In the horizontal plane, the relative strain is equal to $\varepsilon_1$, and the elementary compression force is equal to

$$dP_{обж} = \varepsilon_1 \ E \ dh_{сж}. \quad (16)$$

Relative deformations in the horizontal and vertical planes $\varepsilon_1$, and $\varepsilon_{сж}$ are interrelated via the Poisson's ratio $\mu$.

$$\varepsilon_1 = \mu \varepsilon_{сж}. \quad (17)$$

Then, substituting (10) in (17), we get

$$\varepsilon_1 = \mu \frac{h_{сж(x)}}{h}. \quad (18)$$

The elementary force acting on the side of the horizontal column is equal to

$$dP_{обж} = \mu \frac{h_{сж(x)}}{h} E \, dh_{сж}, \quad (19)$$

well, the force that compresses the oblique chamfer of the blade

$$P_{обж} = \mu \frac{E}{2} \frac{h_{сж}^2}{h}. \quad (20)$$

Taking into account that the Poisson's ratio has small values, we can say that $P_{обж}$ is an insignificant fraction of the value of $P_{сж}$ and substituting the values of all the forces that counteract $P_{кр}$ in the expression (8), we get

$$P_{кр} \geq \delta\sigma_p + \frac{E}{2}\frac{h_{сж}^2}{h} tg\beta + \frac{f\mu E}{2}\frac{h_{сж}^2}{h} + f\left(\frac{Eh_{сж}^2}{4h} tg\beta \sin 2\beta + \frac{\mu E}{2}\frac{h_{сж}^2}{h}\cos^2\beta\right) \quad (21)$$

or, mathematically transforming, we will bring to the final form

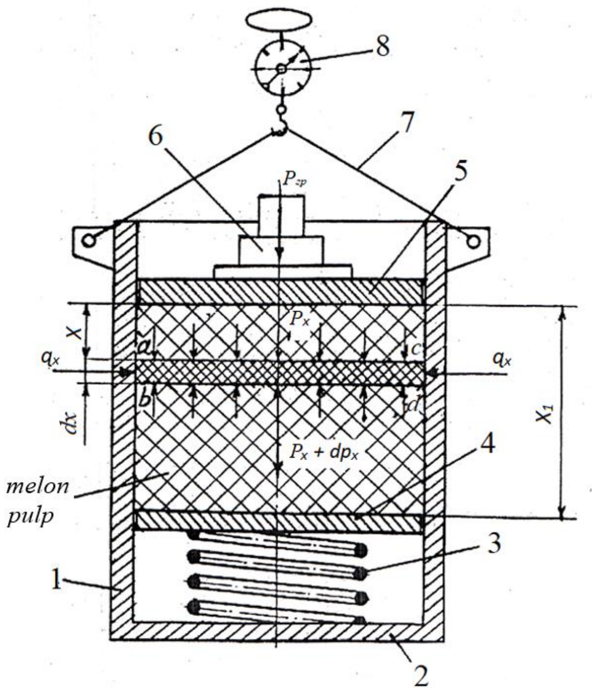
$$P_{кр} = \delta\sigma_p + \frac{E}{2}\frac{h_{сж}^2}{h}\left[tg\beta + f\sin^2\beta + \mu f(1+\cos^2\beta)\right], \quad (22)$$

where $\sigma_p$ – destructive contact stress at the edge of the blade;
$h$ and $h_{сж}$ – accordingly, the thickness of the material layer to be cut and the layer compressed by the blade before the start of cutting;
$\beta$ – blade sharpening angle;
$E$ – modulus of elasticity;
$\mu$ – Poisson's ratio;
$f$ – the coefficient of friction of the material on the blades.

In the derived expression (22), the values ($\beta$ and $\delta$) are the embedded parameters of the blade, ($h$, $h_{сж}$) are the mode parameters of processing, $(E,\ \mu,\ f,\ \sigma_p)$ – the determined physical and mechanical properties of the material.

**RESULTS OF THE STUDY.**

Determination of the Poisson's ratio by the method of E. M. Gutiard. The method is based on the determination of the volume shrinkage of the test sample of plant material on a laboratory installation, shown in Fig.2 [2].




1-body; 2-bottom; 3-compression spring; 4-false bottom; 5-piston;
6-load; 7-suspension; 8-dynamometer

**Fig. 2. Laboratory installation for studying the shrinkage of plant material**

Experimental studies are carried out as follows. Prepare the melon pulp cut into cubes (slices), fill the cylindrical container 1 to the upper edge and weigh the mass of the test sample with a dynamometer 8. Then, a piston 5 is placed on top of the sample, the initial height of the sample layer is fixed (the measuring ruler is not shown in Fig.2) and, placing disequilibrium loads 6 on the piston, an axial compression is created. As the load gradually increases, the compression force $P_x + dp_x$ and the volume shrinkage value *dx* are recorded. By calculating the cross section of the container by the following expression

$$S = \frac{\pi}{4} d^2 = 0,785 d^2, \tag{23}$$

calculate the strain stress

$$\sigma = \frac{P_x + dp_x}{S} = \frac{P_x + dp_x}{0,785 d^2}. \tag{24}$$

As the compression pressure increases, the axial force from the weight of the sample itself $P_x$, the load $dp_x$, and the lateral pressure $q_x$ from the friction forces on the side surface act in the material. To simplify the problem under consideration, we assume that the lateral pressure $q_x$ is compensated by the stiffness of the compression spring 3.

According to the fixed values of the volume shrinkage and the calculated values of the strain stress, a graph of the dependence is plotted (Fig. 3). In this case, the modulus of elasticity is determined by the following formula

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1}, \tag{25}$$

where $\sigma_1$ and $\sigma_2$ – stress before deformation and after the corresponding axial compression;

$\varepsilon_1$ и $\varepsilon_2$ – accordingly, the modulus of deformation before and after axial compression.

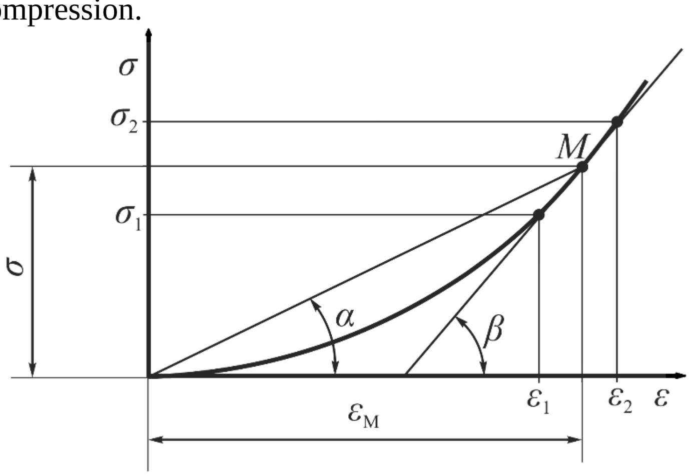



**Fig. 3. Graph of the dependence of the modulus of deformation on the modulus of elasticity**

By drawing the tangent through the averaged point of the deformation modulus, the tangent of the slope angle, which characterizes the elastic modulus, is determined. This method can also be used to determine the Poisson's ratio $\mu$.

To derive the equation that determines the Poisson's ratio, we use a mathematical device. We will select the element *abcd* in the pressed material in the chamber, which is affected by the axial forces $P_x$, $P_x + dp_x$ and the lateral pressures $q_x$. The ratio of the relative longitudinal and transverse deformations of the selected element is determined by the equation

$$\frac{q_x}{E} = \mu \frac{P_x}{E} + \mu \frac{dp_x}{E} \tag{26}$$

or

$$q_x = P_x \frac{\mu}{1-\mu}. \tag{27}$$

The equilibrium condition of the *abcd* element is defined by the expression

$$P_x ab - 2\,fq_x a\,d\,x - 2\,fq_x b\,d\,x - \left(P_x + dp_x\right)ab = 0\big), \tag{28}$$

or

$$dP_x = -2\,fq_x \frac{a+b}{ab} dx\ . \tag{29}$$

Substituting the values of $q_x$ from (26) to (29) we get

$$\frac{dP_x}{P_x} = -2\frac{\mu}{1-\mu}\frac{a+b}{ab}\,fdx\ . \tag{30}$$

Integrating, we get

$$\ln P_x = -2\frac{\mu}{1-\mu}\frac{a+b}{ab}\,fx + c\,. \tag{31}$$

For $x=0$; $P_x=P_0$, we get that $c=lnP_0$. Substituting this value with c, we get $P_x = P_0\ e^{-A_x}$

where

$$A_x = 2\,f\,\frac{\mu}{1-\mu}\frac{a+b}{ab}. \tag{32}$$

At $x=x_1$, we get the pressure on the bottom of the false bottom $P'$. Taking into account (32), we get

$$\frac{\mu}{1-\mu}=\frac{\ln\dfrac{P'}{P_0}}{2f\,\dfrac{a+b}{ab}\,x_1}\ . \qquad (33)$$

Denoting $2f\,\dfrac{a+b}{ab}=A'$, we find the value $\mu$:

$$\mu=\frac{\ln(P'/P_0)}{A'x+\ln(P'/P_0)}=\frac{1}{\dfrac{A'x_1}{\ln(P'/P_0)}+1}. \qquad (34)$$

Revealing the value of the constant coefficient, we get

$$\mu=-\frac{1}{2f\,\dfrac{a+b}{ab}\,x}\cdot\frac{ab}{\ln(P'/P_0)}+1. \qquad (35)$$

If $x$ - is the current height of the test layer, then

$$x=\delta\,\frac{\gamma_0}{\gamma}, \qquad (36)$$

where $\gamma -$ is the current density.

The pressure on the false bottom of the chamber is equal to

$$P_{x_1}=c\gamma^m-c_1\gamma^{m_1}, \qquad (37)$$

well, the pressure on the piston is equal to the weight of the load

$$P_0=c\gamma^m, \qquad (38)$$

where $c_1\gamma^{m_1}$ – friction force per 1 $m^2$ of the transverse area of the chamber.

After substituting the above values into the expression (35) and converting, we get

$$\mu=-\frac{1}{2f\,\dfrac{a+b}{ab}\,x}\cdot\frac{ab}{\ln(1-\dfrac{c_1}{c})}+1. \qquad (39)$$

This expression takes into account the change in the axial and lateral forces on the height of the container acting on the test material. Given that the axial and lateral pressures are constant, it is possible to convert (39) to a simpler expression to determine the Poisson's ratio

$$\mu=\frac{1}{c_0^{m-m_1-1}+1}. \qquad (40)$$

Thus, the determined values of the elastic modulus and the Poisson's ratio $\mu$ in formula (22) provide a basis for the theoretical determination of the critical cutting force. However, the exact value of the local modulus $E_л$ of deformation corresponding to the moment of compression of the material by the edge of the cutting tool blade can be obtained on the basis of laboratory studies.

## CONCLUSIONS

The determined values of the elastic modulus and the Poisson's ratio provide a basis for the theoretical determination of the critical cutting force, but to determine the exact value of the local modulus of deformation corresponding to the moment of compression of the material by the edge of the cutting tool blade, laboratory studies are required.

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