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DESIGN OF CONTROLLERS FOR SEXTUPLE TANK PROCESS

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Abstract

This work focuses on the development of controllers for the six spherical tank interacting process. The presence of interaction between the tanks and the dead time makes the control of sextuple tank process more interesting and challenging and it is ideally suited to demonstrate a multivariable level control problem. Also Control of liquid level in a spherical tank is important because the process is highly nonlinear due to the variation in the area of cross section of the system with height. In this paper decentralized PI, optimal PI and Fractional order – Proportional Integral (FO-PI) controllers are designed to control the level of the sextuple tank process. The simulation results show that FO-PI controller gives better performance comparatively.

Key words: Sextuple tank process, Fractional order controller, Decentralized PI, Optimal PI.

Introduction

Most of the industrial processes are basically Multi-Input/ Multi-Output (MIMO) systems. In Single Input Single Output (SISO) system, the primary objective is to maintain only one variable nearer to its set point, though several measured variables involved (e.g. cascade and feed forward control). By contrast, multivariable control involves the objective of maintaining several controlled variables at independent set points [1]. The multivariable process called the sextuple tank process consists of six spherical tanks interacting to each other, two control valves, one recycle tank and one pump [2]. This process presents a high degree of nonlinearity and a RHP transmission zero which can be moved from one side of the complex plane to the other side by changing the valve positions x_1 and x_2. This process is ideally suited to illustrate many concepts in multivariable control. Spherical tanks find wide application in gas plants.

$$A_4 \frac{dh_4}{dt} = f_4 = x_2.F_2 - R_4\sqrt{h_4}$$
$$A_5 \frac{dh_5}{dt} = f_5 = R_4\sqrt{h_4} - R_5\sqrt{h_5}$$
$$A_6 \frac{dh_6}{dt} = f_6 = (1-x_1).F_1 + R_5\sqrt{h_5} - R_6\sqrt{h_6}$$

The tuning methods of multi-loop controller for a multivariable process are discussed in [1]. The effective and time saving tool for robust lower order multivariable controller design for sextuple tank process is discussed in [2]. The control aspects of spherical tank using Internal Model based Controller (IMC) PI tuning setting in real time is dealt in [3]. The model based tuning for spherical tank process with time delay is used in [4] and also Smith Predictor controller is designed for spherical tank process. Simulated annealing tuned PI controller tuning for a spherical tank process is given in [5]. The PID controller design for a TITO system is proposed [6].

The multivariable zero dynamics of the system can be made both minimum phase and non minimum phase by simply changing the valve [7]. The controller tuning for Quadruple-Tank Process (QTP) is discussed [8].

The tuning rules for Optimal PI/PID controllers and fractional order PID controllers are given in [9]. The design of FO-PI controller for liquid level control system is discussed in [10]. A comparative introduction of four fractional order controllers is discussed in [11]. The fractional order control basics are discussed in [12].

This paper is organized as follows. The physical model of the sextuple tank process is presented in section II. Section III discusses the Relative Gain Array analysis to determine the best controller pairing. Section IV consists of the controller design for the sextuple tank process. Section V discusses the simulation results along with the detailed comparative analysis. Finally section VI concludes the paper.

Process Description

The schematic diagram of the process is shown in the Figure 1 [2]. The objective is to control the levels h_3 and h_6 manipulating the two valves defining the flow rates F_1 and F_2 and the valve distribution flow factors of these flow rates ($0 \le x_1 \le 1$, $0 \le x_2 \le 1$) that distribute the total feed among the tanks. The simplified model explained by (1) is developed for this process [2].

$$A_1 \frac{dh_1}{dt} = f_1 = x_1.F_1 - R_1\sqrt{h_1}$$

$$A_2 \frac{dh_2}{dt} = f_2 = R_1\sqrt{h_1} - R_2\sqrt{h_2}$$

$$A_3 \frac{dh_3}{dt} = f_3 = (1-x_2).F_2 + R_2\sqrt{h_2} - R_3\sqrt{h_3} \tag{1}$$

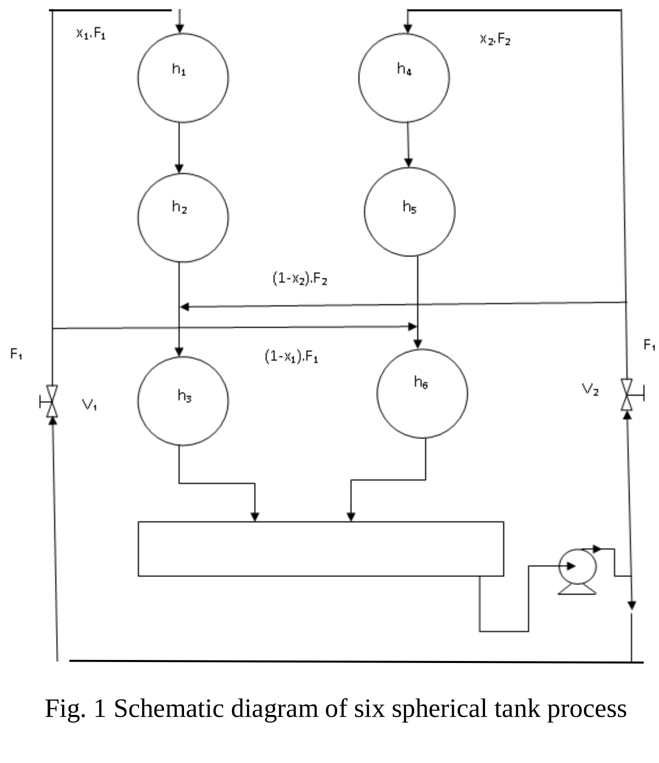

Fig. 1 Schematic diagram of six spherical tank process

After linearizing the model and transforming into laplace domain at the operating points the corresponding transfer function matrix is given by (2) [2].

$$\begin{bmatrix} h_3(s) \\ h_6(s) \end{bmatrix} = \begin{bmatrix} \dfrac{x_1 c_1 e^{-0.9s}}{\prod_{i=1}^{3}(\tau_i s+1)} & \dfrac{(1-x_2)c_1 e^{-0.3s}}{\tau_3 s+1} \\ \dfrac{(1-x_1)c_2 e^{-0.3s}}{\tau_6 s+1} & \dfrac{x_2 c_2 e^{-0.9s}}{\prod_{i=4}^{6}(\tau_i s+1)} \end{bmatrix} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \tag{2}$$

with

$$c_1 = \frac{2\sqrt{h_{3s}}}{R_3}, c_2 = \frac{2\sqrt{h_{6s}}}{R_6}$$
$$\tau_i = \frac{2A_i\sqrt{h_{is}}}{R_i} \tag{3}$$

where

$$A_i = \pi(D_i h_i - h_i^2) \text{ and } R_i = a_i\sqrt{2g} \tag{4}$$

g = gravitational constant

a_i = cross sectional area of the discharge pipe from the tank i

D_i = diameter of the tank i

When the sum of x_1 and x_2 is greater than one, the system has a RHP-zero. If $x_1+x_2=1$, the system has a zero located at the origin and as greater goes this sum, the zero is moved away of the origin along the positive axis.

The parameters of the sextuple tank process and the chosen operating points are given in Table 1 and 2 respectively [2].

Table I: Process Parameters

Parameters	Value
D1, D4 [cm]	35
D2, D5[cm]	30
D3, D6[cm]	25
R1, R4[$cm^{2.5}$ min^{-1}]	1690
R2, R5[$cm^{2.5}$ min^{-1}]	1830
R3, R6[$cm^{2.5}$ min^{-1}]	2000

Table II: Operating Points

Variables	Operating point
h_{1s}, h_{4s} [cm]	2.75,2.02
h_{2s}, h_{5s}[cm]	2.34,1.72
h_{3s}, h_{6s}[cm]	4.84,3.24
F1,F2[L/min]	4,4
x_1,x_2	0.7,0.6

After substituting the parameters and the operating points, the transfer function matrix obtained is given by (5).

$$\begin{bmatrix} h_3(s) \\ h_6(s) \end{bmatrix} = \begin{bmatrix} \frac{1.54*10^{-3}e^{-0.9s}}{(0.545s+1)(0.34s+1)(0.674s+1)} & \frac{8.8*10^{-4}e^{-0.3s}}{0.674s+1} \\ \frac{5.4*10^{-4}e^{-0.3s}}{0.399s+1} & \frac{1.08*10^{-3}e^{-0.9s}}{(0.352s+1)(0.218s+1)0.399s+1)} \end{bmatrix} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \quad (5)$$

Here G_{11} and G_{22} are of third order and since first order plant system with time delay is required for computing decentralized IMC, FO-PI and optimal PI Controller tuning parameter, a MATLAB file called opt_app.m is used to approximate third order by first order transfer function and the approximated transfer function matrix given by (6) is used to design the controller tuning parameters.

$$\begin{bmatrix} h_3(s) \\ h_6(s) \end{bmatrix} = \begin{bmatrix} \frac{1.54*10^{-3}e^{-1.46s}}{(1.0599s+1)} & \frac{8.8*10^{-4}e^{-0.3s}}{0.674s+1} \\ \frac{5.4*10^{-4}e^{-0.3s}}{0.399s+1} & \frac{1.08*10^{-3}e^{-1.25s}}{(0.656s+1)} \end{bmatrix} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \quad (6)$$

The model and control of the sextuple tank process are studied at the operating points given in Table 2 at which the system will be shown to have non minimum phase characteristics with RHP-zero at 1.0246 [2].

Relative Gain Array

Interaction analysis of multivariable system has been an important issue for control structure design (such as input output pairing) and decentralized control problems [13]. The first quantitative measure of interaction was the Relative Gain Array (RGA) introduced by Bristol [14]. It has been used widely and successfully in process industries [15, 16]. The most well known results on the RGA are that a plant with large or negative elements in the RGA is difficult to control and that input and output variables should be paired such that the diagonal elements of the RGA are as close as possible to unity [17, 18]. When the number of inputs and outputs are high an alternative method called the steady –state interaction indices developed by Chang and Davison [19] provide more accurate analysis of multiloop interaction.

In this paper, the pairing of the loops is decided by the Relative Gain Array (RGA) [20] analysis. An important advantage of the RGA method is that it requires minimal process information: namely, steady state gains. Another advantage is that the results are independent of both the physical units used and the scaling of the process variables. For non minimum phase settings of the sextuple tank process λ is 1.40. So u_1 must be paired with y_1 and u_2 must be paired with y_2 for better performance.

Design of Controllers

A. Design of decentralized PI controller

The basic block diagram of multiloop control structure with two PI controllers G_{c1} and G_{c2} is shown in Fig. 2 and its closed loop equation can be written in matrix form as given in (7) [6].

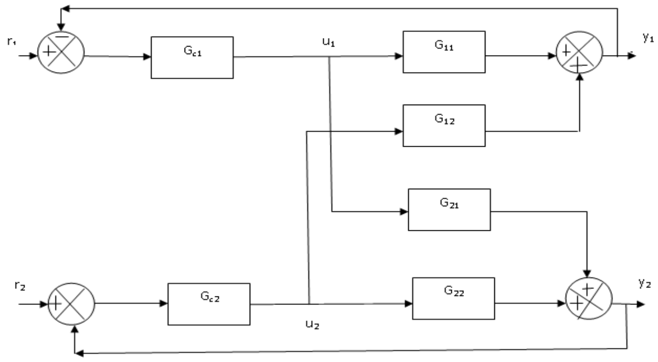

Fig. 2 Basic block diagram of multi loop control structure

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} G_{c1} & 0 \\ 0 & G_{c2} \end{bmatrix} \begin{bmatrix} r_1 - y_1 \\ r_2 - y_2 \end{bmatrix} \quad (7)$$

The decentralized PI control structure includes two PI SISO controllers. For designing decentralized controller Skogested IMC method is used [21]. Therefore closed loop time constant is equal to the time delay of the system. The decentralized PI controller parameters are given in the Table III below.

Table III Decentralized PI Controller Parameters

Controller	Gain (K_c)	Integral time constant (T_i)
Decentralized PI	K_{c11}=236 K_{c22}=244	T_{i11}=1.0598 T_{i22}=0.656

B. Design of FO-PI controller

The decentralized FO-PI control structure includes two FO-PI SISO controllers. The structure is shown in Fig. 3

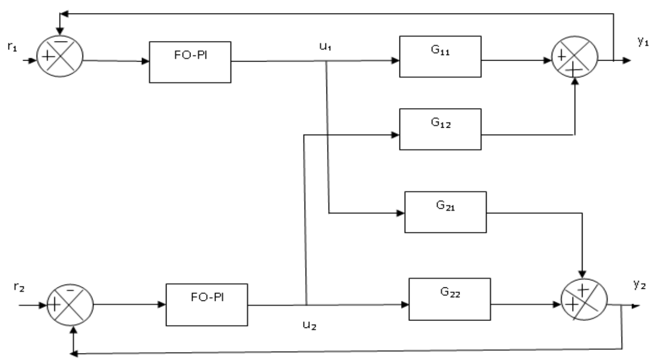

Fig. 3 Decentralized control structure with two FO-PI controllers

The FO-PI controller in frequency domain is simply written as [10]

$$C(s) = K_p + \frac{K_i}{s^{\alpha}} \tag{8}$$

where K_p and K_i are the proportional and integral gain values

of the fractional controller and α is the non integer order of the fractional integrator. Tuning the gains K_p, K_i and non integer order α is discussed in [22] and [23] experimentally validates the tuning rules.

The tuning rules [22] are given by

$$K_p = \frac{0.2978}{K(\tau + 0.000307)} \tag{9}$$

$$K_i = \frac{K_p(\tau^2 - 3.402\tau + 2.405)}{0.8578T} \tag{10}$$

$$\alpha = \begin{cases} 0.7 & if \quad \tau < 0.1 \\ 0.9 & if \quad 0.1 \le \tau < 0.4 \\ 1.0 & if \quad 0.4 \le \tau < 0.6 \\ 1.1 & if \quad \tau \ge 0.6 \end{cases} \tag{11}$$

These tuning rules are based on Fractional Maximum Sensitivity Constrained Integral Gain Optimization method (F-MIGO) for generic First Order Plus Delay Time (FOPDT) model and the relative delay is given by

$$\tau = \frac{L}{L+T} \tag{12}$$

The decentralized FO-PI controller parameters are given in the Table IV below.

Table IV Decentralized FO-PI Controller Parameters

Controller	Gain (K_c)	Integral time constant (T_i)	α
Decentralised FO-PI	K_{c11}=333 K_{c22}=419	T_{i11}=1.18 T_{i22}=0.93	1 1

C. Design of optimal PI controller

The optimal tuning rule for PI controller is based on the minimization of the integrated absolute error (this yields low overshoot and a low settling time at the same time) subject to a constraint on maximum sensitivity in order to provide a required level of robustness [9]. The following structure for the controller parameters has been used.

$$k_p = \frac{1}{k}(a\tau^b + c)$$

$$T_i = T(a(\frac{L}{T})^b + c) \tag{13}$$

The values of a, b and c for K_p and T_i are given in Table V [9].

Table V K_p tuning rule parameters for a PI controller

Control task	Maximum sensitivity M_s=1.4		
	a	b	c
K_p	0.3220	-1.049	-0.2292
T_i	0.1726	1.156	0.9907

The optimal PI controller parameters are given in the Table VI below.

Table VI Optimal PI Controller parameters

Controller	Gain (K_c)	Integral time constant (T_i)
Optimal PI	K_{c11}=222 K_{c22}=253	T_{i11}=1.315 T_{i22}=0.888

Results and Discussion

Simulations are performed using MATLAB simulink for decentralized PI, FO-PI and Optimal PI controllers to validate their performances. The decentralized PI, FO-PI and optimal PI controller closed loop responses of tank 3 and 6 are given below in Fig.4 and 5. The performance index criterions such as Integral Square Error (ISE), Integral Absolute Error (IAE) and Integral Time Absolute Error (ITAE) for all the controllers for the level output of tank 3 and 6 are tabulated in the Table VII and VIII respectively.

Figures 6 and 7 shows the servo responses of various controllers for the levels h3 and h6 of the sextuple tank process respectively. A step change of +50% is given at the 30th second and a negative step change of -50% is given at the 60th second. Figures 8 and 9 shows the regulatory responses of various controllers. A load disturbance of +10% is applied at the 40th second and the performance index criterions for level h_3 and h_6 are compared and presented in Table IX and X

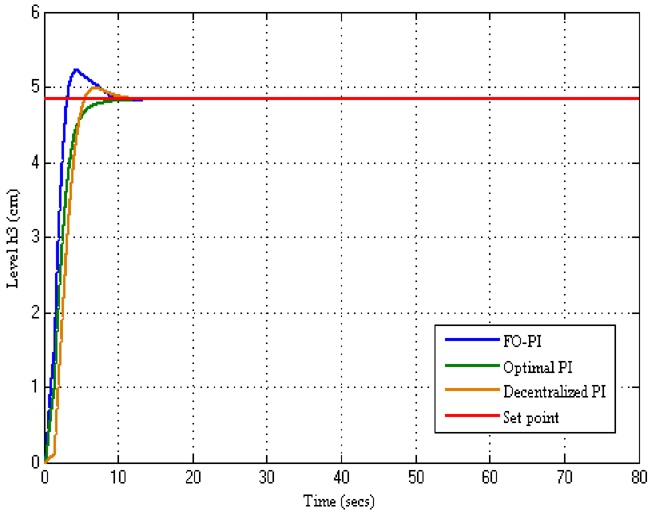

Fig. 4 Closed loop step responses for level h_3 of the sextuple tank process for the set point of 4.84cm with different controllers.

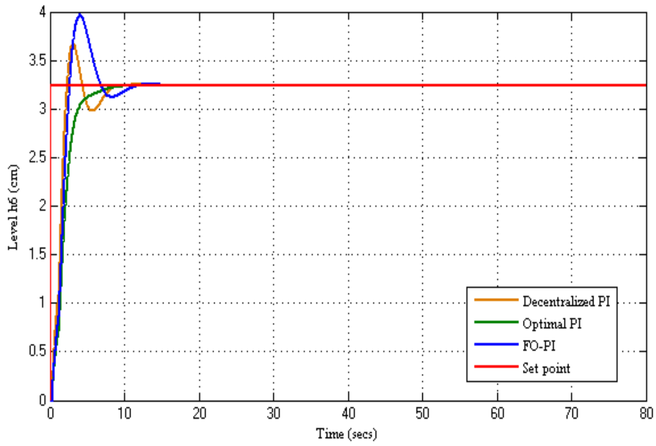

Fig. 5 Closed loop step responses for level h_6 of the sextuple tank process for the set point of 3.24cm with different controllers.

Table VII Performance index comparison for level h_3 with different controllers for the sextuple tank process.

Per.index / controller	ISE	IAE	ITAE
Decentralised PI	54.91	14.72	27.47

Optimal PI	39.35	11.84	20.38
FO-PI	30.75	9.8	17.12

Table VIII Performance index comparison for level h_6 with different controllers for the sextuple tank process.

Per.index / controller	ISE	IAE	ITAE
Decentralised PI	13.57	7.21	15.78
Optimal PI	14.42	6.693	10.19
FO-PI	11.07	5.59	7.506

As seen from the simulation results for level h_3 and h_6 of the sextuple tank process, the FO-PI controller has the least ISE, IAE and ITAE values compared to the decentralized PI controller and optimal PI controller.

Conclusion

Thus the control of liquid levels h_3 and h_6 for the six spherical tank interacting process with non minimum phase behavior is discussed in this paper. Decentralized PI, FO-PI and Optimal PI controllers are designed to control the levels h_3 and h_6 of the sextuple tank process. ISE, IAE and ITAE values are calculated to find the performance of the controllers. The performance index criteria ISE, IAE and ITAE values is always less for FO-PI controller compared to decentralized PI controller and optimal PI Controller. The effectiveness of the controller is tested in simulation.

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