

## **Effects of Heat Absorption and Chemical Reaction on MHD Nanofluid Flow and Heat Transfer Past a Stretching Sheet with Melting**

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### **Abstract**

An investigation is made on a steady two dimensional boundary layer flow of a viscous, incompressible, electrically conducting, heat absorbing and chemically reacting nano-fluid over a stretching sheet, with melting, in the presence of an applied transverse magnetic field taking viscous and joule dissipations into account. Using adequate similarity transformations non-linear partial differential equations, governing to the problem, are transformed into non-linear ordinary differential equations and then solved using bvp4c (Matlab's boundary value problem solver). Numerical results of velocity, temperature, and species concentration are depicted graphically. Numerical values of skin friction coefficient, local Nusselt number, local Sherwood number are presented in tabular form

### **Introduction**

Melting heat transfer has wide industrial applications such as casting, welding and magma solidification, permafrost melting and thawing of frozen ground, etc. Many research papers have been published on the effect of melting heat transfer. Accordingly, Tien and Yen [1] have examined "The effect of melting on forced convection heat transfer". Their study indicates that melting retards the rate of heat transfer. Then after, Epstein and Cho [2] have studied "Melting heat transfer in steady laminar flow over a flat plate". The result shows that melting inhibits the heat transfer rate and reduces the local Nusselt number. Further, Ishak et al. [3] examined "Melting heat transfer in steady laminar flow over a moving surface". They indicated that melting decreases the local Nusselt number at the solid-liquid interface. A comprehensive study is made by many researchers [4-9].

A pioneer work entitled "Convective transport in nanofluids" has been done by Buongiorno [10]. In his model, he has considered the influences of Brownian and thermophoresis diffusions. After this pioneer work, many papers have been published on nanofluid by employing the same model. For example, scholars, Khan and Aziz [11], Kuznetsov and Nield [12], Khan and Pop [13], Yacob et al. [14], Makinde and Aziz [15] examined the flow of a nanofluid past a stretching surface under different circumstances. All the above studies signify that both Brownian motion and thermophoresis reduces the heat transfer rate of the laminar flow. Besides, Olanrewaju and Makinde [16], Khan and Reddy [17], Vajravelu et al. [18] analyzed the stagnation point flow of a nanofluid past stretching surface with convective boundary condition. The findings reveal that the momentum and energy boundary layer thickness are antagonistic. Furthermore Noor et al. [19] have examined the "Mixed

convection stagnation flow of a micropolar nanofluid along a vertically stretching surface with slip effects”.

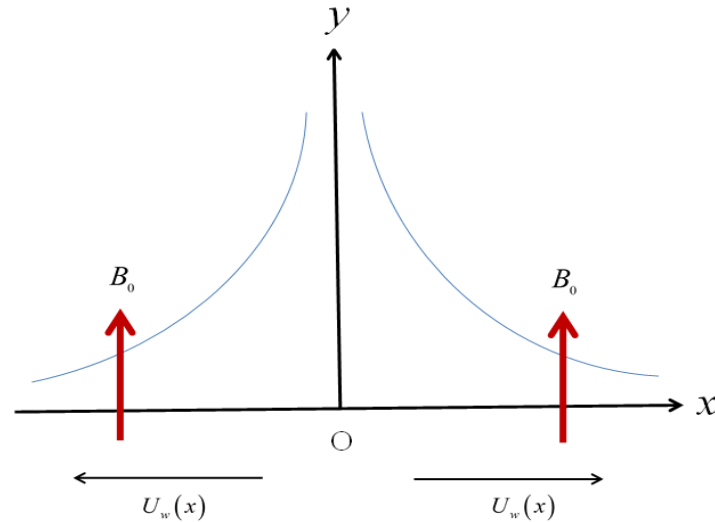
Dissipative effects are one of the important factors in many fluid engineering devices. It has been seen that there is a significant heat transfer to the fluid due to the energy dissipation. Kameswaran et al. [20] investigated the “Hydromagnetic nanofluid flow due to a stretching or shrinking sheet with viscous dissipation and chemical reaction effect”. To describe the nanofluid flow, they used the model “nanoparticle volume fraction”. Hady et al. [21] studied “Radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet”. Khan et al. [22] carried out an investigation on “Unsteady MHD free convection boundary-layer flow of a nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects”. There are several industrial situations where the surface is convectively heated by external source. This causes a certain change in the surface temperature gradient and affects the temperature of the fluid within the boundary layer. Makinde and Aziz [23] investigated “Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition”. In their governing transport equation, they included Brownian motion and thermophoresis effects. Mahatha et al. [24] studied “Dissipative Effects in Hydromagnetic Boundary Layer Nanofluid Flow past a Stretching Sheet with Newtonian Heating”.

Though the considerable amount of work has been done in the nano-fluid flow over a stretching surface, still more attention is needed to study the effects of viscous and joule dissipations on melting heat transfer of a heat absorbing and chemically reacting nano-fluid flow past a stretching sheet. Hence authors were motivated to investigate the effects of heat absorption and chemical reaction on MHD nanofluid flow and heat transfer past a stretching sheet with melting considering viscous and joule dissipations into account. Objective of the present paper is to investigate the steady flow of a viscous, incompressible, electrically conducting, heat absorbing and chemically reacting nano-fluid over a stretching sheet, with melting, in the presence of an applied transverse magnetic field taking viscous and joule dissipations into account. The fluid flow model considered in the present study may find applications in high-viscosity fluid engineering devices which work under the influence of an external magnetic field, in fluid flow problems where the cohesive force between the fluid and surface is comparatively larger than the adhesive forces between fluid particles and in flows taking place at high altitudes, in fluid flow problems where the temperature differences between the ambient fluid and flow field is very high and in situations where the whole flow field is heated by some external means.

### **Formulation of the Problem and its Solution**

Consider a steady MHD two-dimensional boundary layer flow and heat transfer of a viscous, incompressible, electrically conducting, heat absorbing and chemically reacting nanofluid past a stretching sheet. Viscous and Joule dissipations have also been taken into consideration. The sheet is melting steadily. The flow is subjected to a constant transverse magnetic field of uniform strength  $B=B_0$  which is applied in the positive  $y$ -direction, normal to the surface. A

coordinate frame has chosen that  $x$ -axis is extending along the stretching sheet and  $y$ -axis is normal to the sheet (figure 1). Temperature of the surface is  $T_m$ , concentration  $C$  take constant value  $C_w$ . The ambient value attained as  $y$  tends to infinity of  $T$  and  $C$  are denoted by  $T_\infty$  and  $C_\infty$ , respectively, while  $T_\infty > T_m$ . The velocity of the stretching sheet is  $u_w = ax$ , where  $a$  and  $b$  are positive constants. Fig. 1 shows the geometry of flow model.



**Fig. 1 Geometry of the problem**

The governing equations for the fluid flow problem are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$(1) u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho_f} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho C_p} u^2 + \frac{Q_0}{\rho C_p} (T - T_m) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - k_1 (C - C_w) \quad (4)$$

where  $\alpha = \frac{k}{(\rho c)_f}$ ,  $\tau = \frac{(\rho c)_p}{(\rho c)_f}$

The boundary conditions are:

$$\left. \begin{aligned} u = u_w = ax, v = 0, T = T_m, C = C_w & \quad \text{at } y = 0 \\ u = v = 0, T \rightarrow T_\infty, C \rightarrow C_w & \quad \text{at } y \rightarrow \infty \\ \alpha \left( \frac{\partial T}{\partial y} \right)_{y=0} = \rho [\lambda + C_s (T_m - T_0)] v(x, 0) & \end{aligned} \right\} \quad (5)$$

where  $u$  and  $v$  are the components of velocity along the  $x$  and  $y$  axes, respectively. Furthermore,  $\nu, \sigma, \rho_f, \rho_p, \alpha, k, (\rho c)_f, (\rho c)_p, \lambda$  and  $C_s$  are respectively the kinematic viscosity coefficient, electric conductivity, density of base fluid, density of nanoparticle, thermal diffusivity, thermal conductivity, heat capacity of the base fluid, heat capacity of the nanoparticle material, latent heat of the fluid, and heat capacity of the solid surface where  $k$  is the thermal conductivity of the fluid,  $\lambda$  is the latent heat of the fluid and  $C_s$  is the heat capacity of the solid surface.

The similarity and dimensionless variables are introduced as follow:

$$\left. \begin{aligned} u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \eta = y \sqrt{\frac{a}{\nu}}, \psi = \sqrt{a\nu} x f(\eta) \\ \theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \phi(\eta) = \frac{C - C_w}{C_\infty - C_w} \end{aligned} \right\} \quad (6)$$

The equation of continuity is satisfied if we choose a stream function  $\psi(x, y)$  such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (7)$$

With the help of above transformations, equation (1) is identically satisfied, and equations (2), (3) and (4) along with boundary conditions (5) take the following forms:

$$f''' + ff'' - f'^2 - Mf' = 0 \quad (8)$$

$$\theta'' + \text{Pr} (f\theta' + Nb\phi'\theta' + Nt\theta'^2 + Ec f'^2 + MEcf'^2 + Q\theta) = 0 \quad (9)$$

$$\phi'' + \text{Le} f\phi' + \frac{Nt}{Nb} \theta'' - Kr\phi = 0 \quad (10)$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} f'(0) = 1, B\theta'(0) + \text{Pr} f(0) = 0, \theta(0) = 0, \phi(0) = 0, \text{ at } \eta = 0, \\ f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 1, \phi(\infty) \rightarrow 1, \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (11)$$

where the governing parameters are defined by:

$$\left. \begin{aligned} M &= \frac{\sigma B_0^2}{\rho_f a}, Le = \frac{\nu}{D_B}, Pr = \frac{\nu}{\alpha}, Q = \frac{Q_0}{a \rho c_p}, Kr = \frac{k_1 \nu}{a D_B}, Nb = \frac{(\rho c)_p D_B (C_\infty - C_w)}{(\rho c)_f \nu}, \\ Nt &= \frac{(\rho c)_p D_B (T_\infty - T_m)}{(\rho c)_f \nu T_\infty}, B = \frac{C_f (T_\infty - T_m)}{\lambda + C_s (T_m - T_0)}, Ec = \frac{a^2 x^2}{C_p (T_\infty - T_m)} = \frac{u_w^2}{C_p \Delta T} \end{aligned} \right\} (12)$$

where  $f'$ ,  $\theta$  and  $\phi$  are the non-dimensionless velocity, temperature and concentration respectively.  $M$ ,  $Le$ ,  $Pr$ ,  $Nb$ ,  $Nt$ ,  $Q$ ,  $B$ ,  $Ec$  and  $Kr$  are respectively, the magnetic parameter, Lewis number, Prandtl number, Brownian diffusion coefficient, thermophoretic diffusion coefficient, heat absorption parameter, melting parameter, Eckert number and chemical reaction rate parameter. Dimensionless melting parameter ( $B$ ) is the combination of Stefan numbers  $\frac{C_f (T_\infty - T_m)}{\lambda}$  (for liquid phase) and  $\frac{C_s (T - T_0)}{\lambda}$  (for solid phase).

Physical quantities which are of much interest in view of the engineering applications, are local skin friction coefficient  $C_f$ , the local Nusselt number  $Nu_x$  and the local Sherwood number  $Sh_x$ . These quantities are defined as,

$$C_f = \frac{\tau_w}{\rho u_w^2}, Nu_x = \frac{x q_w}{k (T_\infty - T_m)}, Sh_x = \frac{x h_m}{D_B (C_w - C_\infty)} \quad (13)$$

where the wall shear stress where the wall shear stress  $\tau_w$ , the wall heat flux  $q_w$  and wall mass flux  $h_m$  are given by

$$\tau_w = \mu \frac{\partial u}{\partial y}, q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}, h_m = -D_B \left( \frac{\partial C}{\partial y} \right)_{y=0} \quad (14) \text{ By using the}$$

above equations, we get

$$C_f \sqrt{Re_x} = -f''(0), \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0), \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0) \quad (15)$$

where  $Re_x$ ,  $Nu_x$ ,  $Sh_x$  are local Reynolds number, local Nusselt number and local Sherwood number, respectively.

### Numerical Procedure

The non-linear ordinary differential equations (8) - (10) with boundary conditions (11) have been solved using by the bvp4c routine of Matlab.

### Results and Discussion

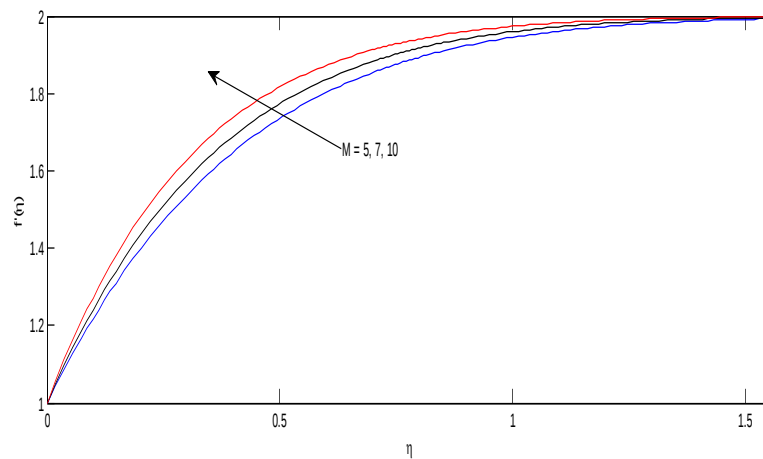
To analyze the effects of flow-parameters viz. velocity ratio parameter  $A$ , magnetic parameter  $M$ , Brownian motion parameter  $Nb$ , thermophoresis parameter  $Nt$ , melting parameter  $B$  and chemical reaction rate parameter  $Kr$ , the profiles of nano-fluid velocity,

nano-fluid temperature and nano particle concentration are depicted graphically in figures 2 to 21 while the values of skin friction coefficient  $-f''(0)$ , Nusselt number  $-\theta'(0)$ , and Sherwood number  $-\phi'(0)$  are tabulated in table 1.

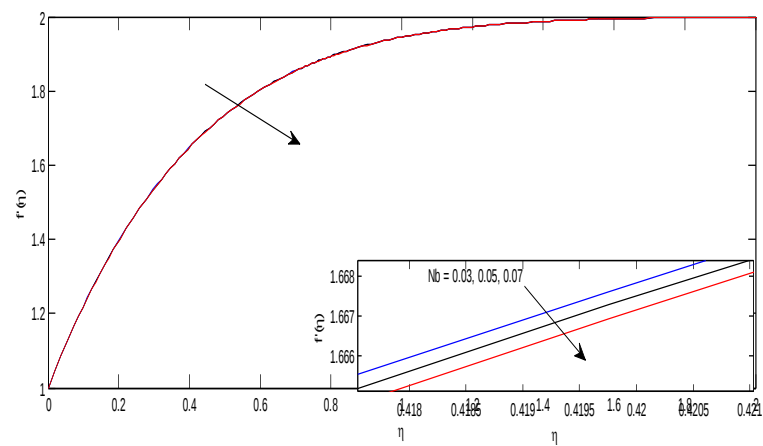
Figures 2 to 8 represent effects of magnetic field, Brownian diffusion, thermophoretic diffusion, melting of the sheet, chemical reaction, heat absorption and Lewis number on the nano-fluid velocity. It is observed from figures 2 to 8 that the magnetic field and chemical reaction have the tendency to enhance the nano-fluid velocity while Brownian motion, Thermophoretic diffusion, melting of the sheet, heat absorption and Lewis number have the reverse tendency on nano-fluid velocity.

Figures 9 to 15 demonstrate effects of pertinent flow parameters on the fluid temperature. It is revealed from figures 9 to 15 that magnetic field, thermophoretic diffusion and heat absorption induces fluid temperature while melting of the sheet and Lewis number reduces the fluid temperature. In the boundary layer region, Brownian diffusion has the tendency to enhance fluid temperature while chemical reaction has reverse effect on it.

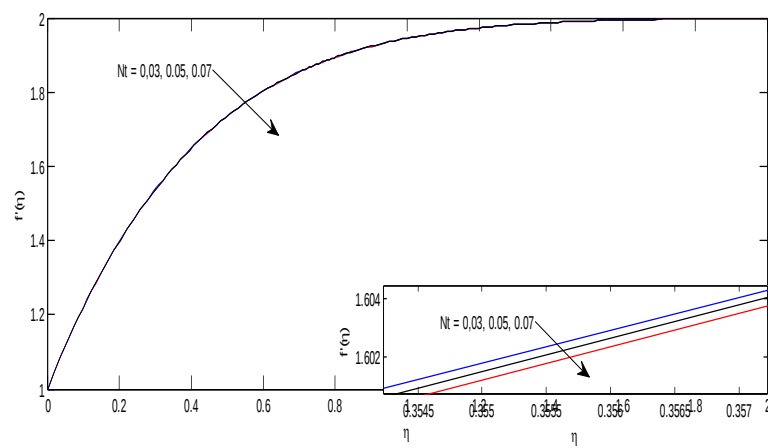
Figures 16 to 21 show the effects of various parameters on fluid concentration. It is evident from figures 16 to 21 that the magnetic field caused for enhancement in fluid concentration while melting of sheet and chemical reaction caused for the reduction in fluid concentration. Furthermore, in the boundary layer region, thermophoretic diffusion induces the fluid concentration whereas Brownian diffusion and Lewis number have reverse effect on it.



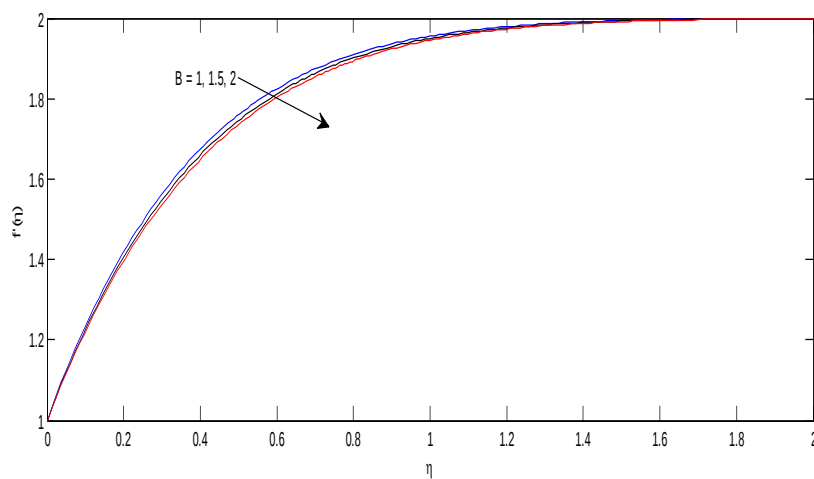
**Figure 2: Velocity profiles for  $M$**



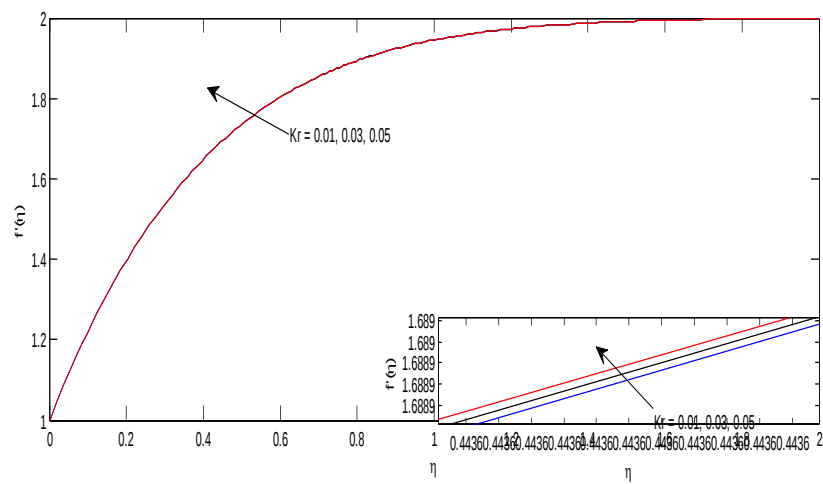
**Figure 3: Velocity profiles for  $Nb$**



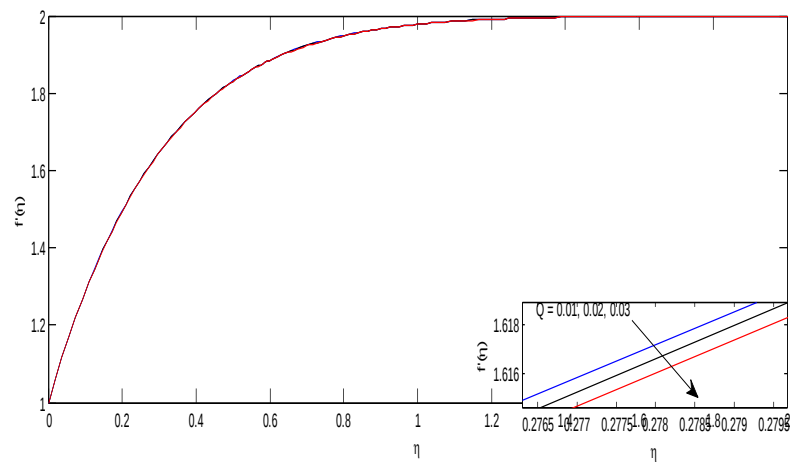
**Figure 4: Velocity profiles for  $Nt$**



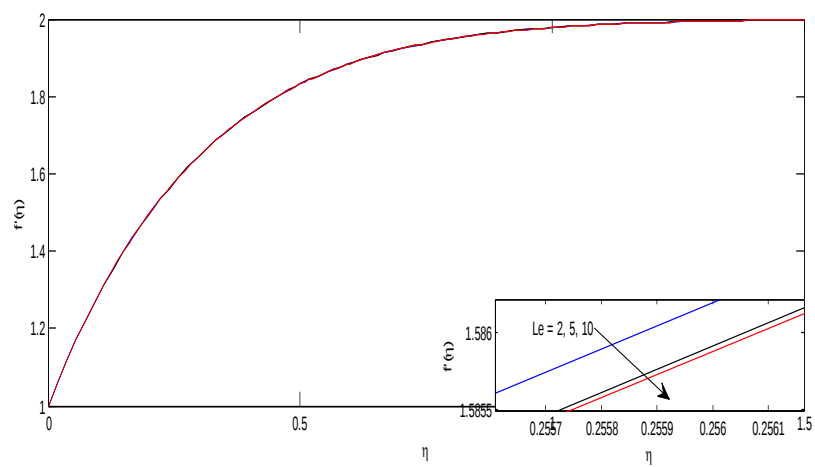
**Figure 5: Velocity profiles for  $B$**



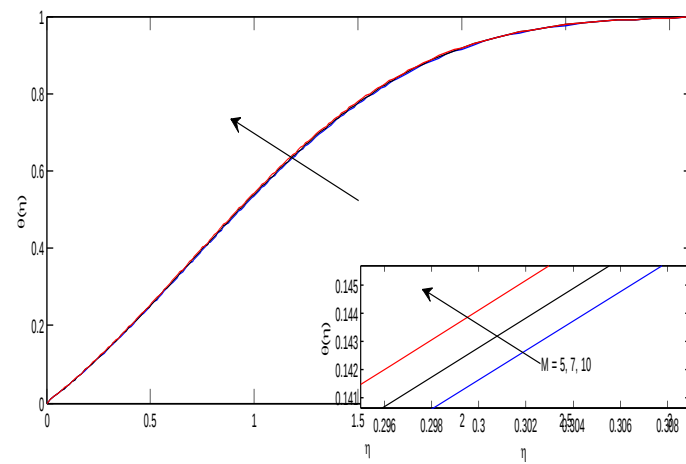
**Figure 6: Velocity profiles for  $Kr$**



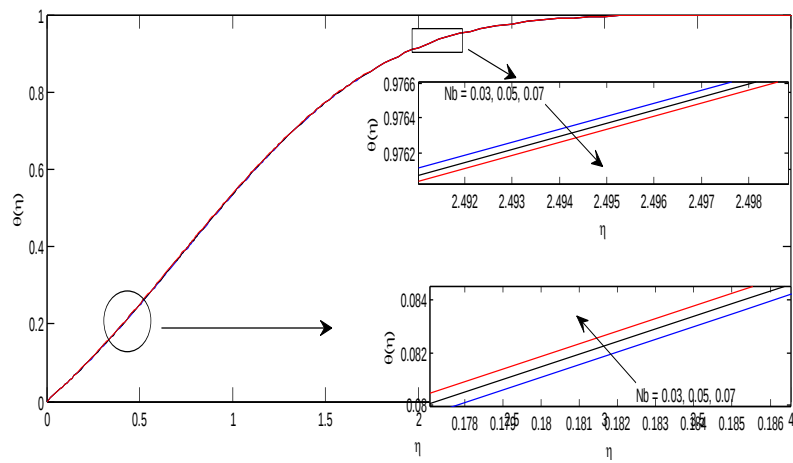
**Fig. 7: Velocity profiles for various value of  $Q$ .**



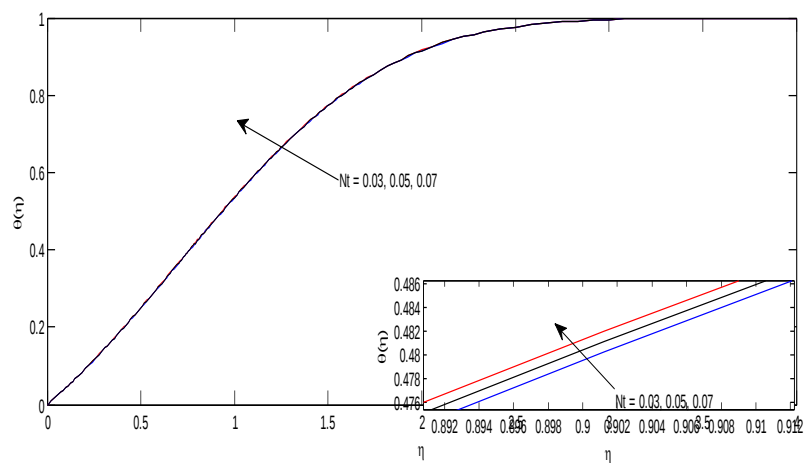
**Fig. 8: Velocity profiles for various value of  $Le$ .**



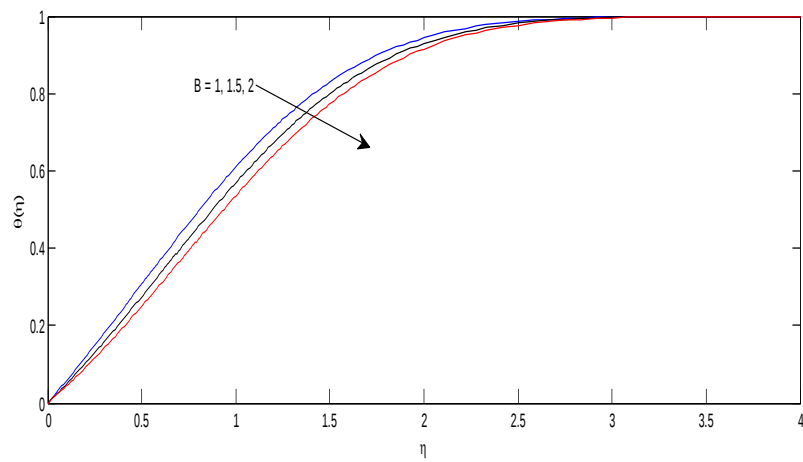
**Figure 9: Temperature profiles for  $M$**



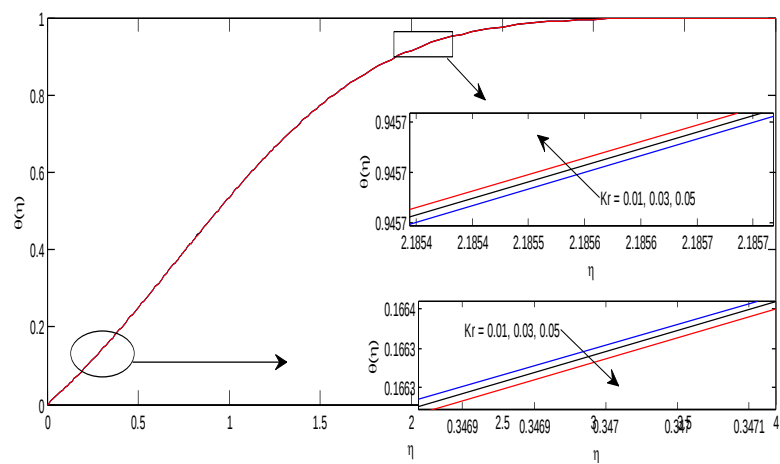
**Figure 10: Temperature profiles for  $Nb$**



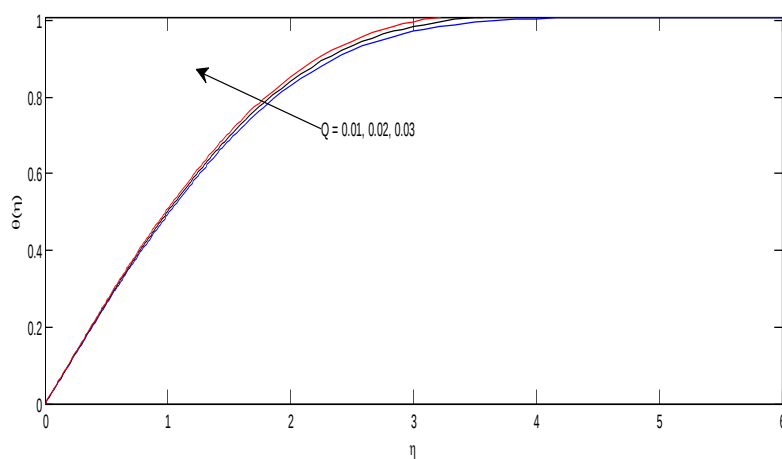
**Figure 11: Temperature profiles for  $Nt$**



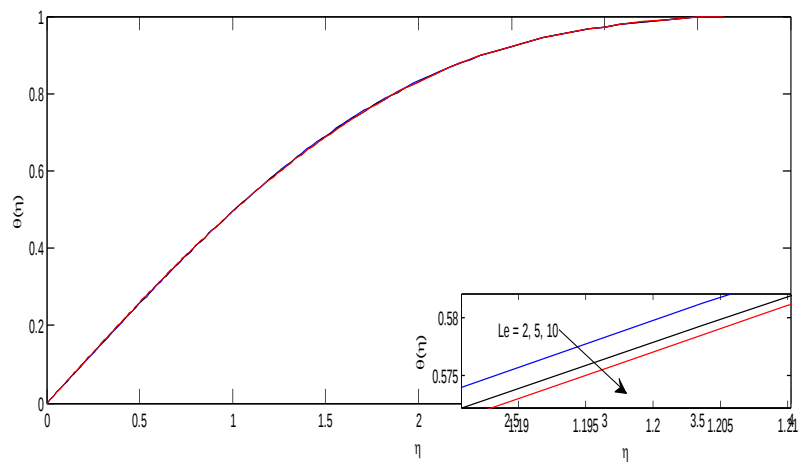
**Figure 12: Temperature profiles for  $B$**



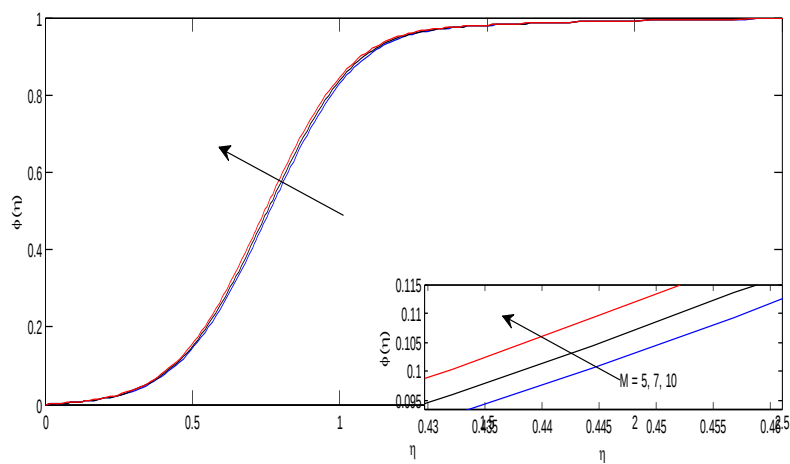
**Figure 13: Temperature profiles for  $Kr$**



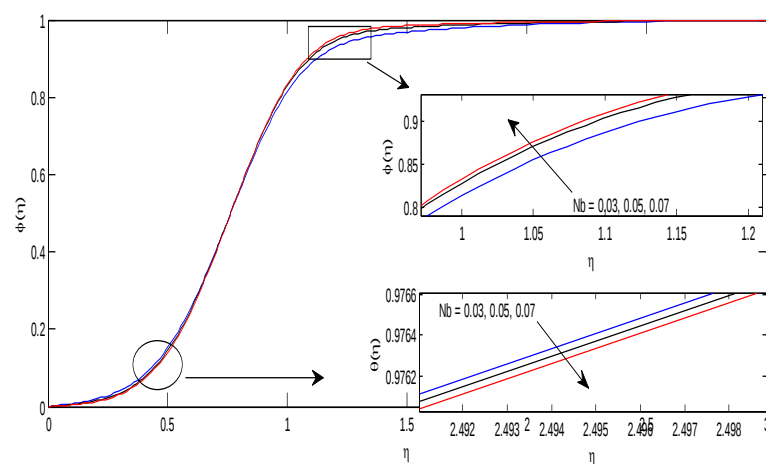
**Figure 14 Temperature profiles for various value of  $Q$ .**



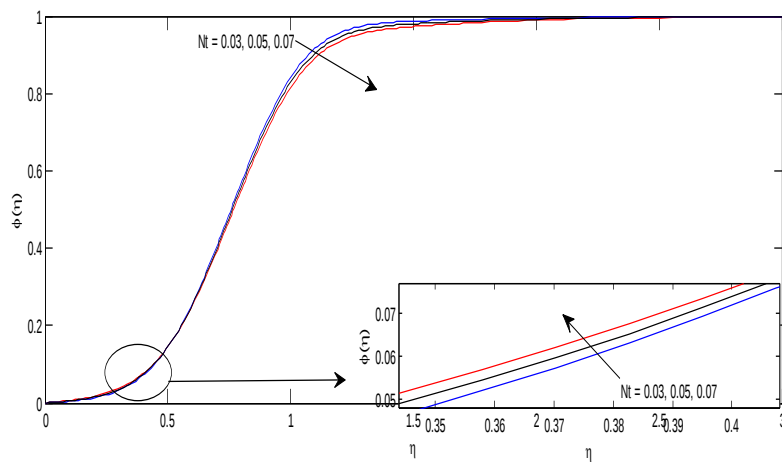
**Figure 15: Temperature profiles for various value of  $Le$ .**



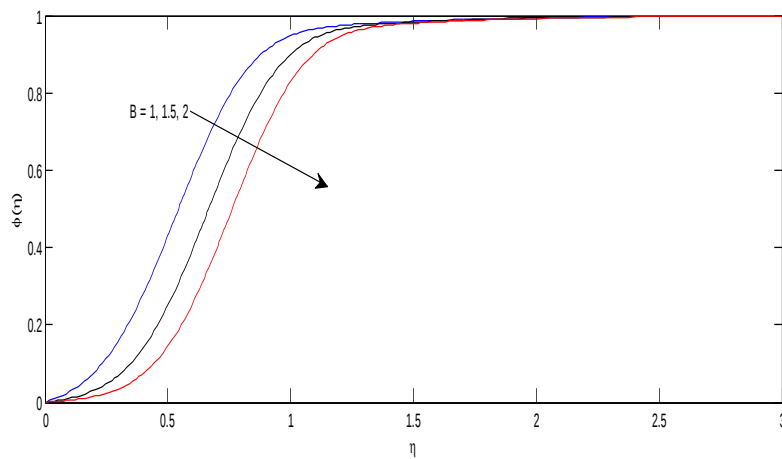
**Figure 16: Concentration profiles for  $M$**



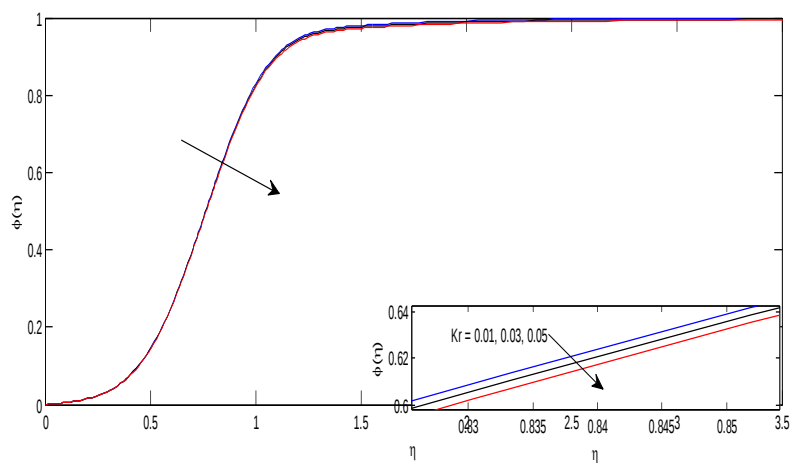
**Figure 17: Concentration profiles for  $Nb$**



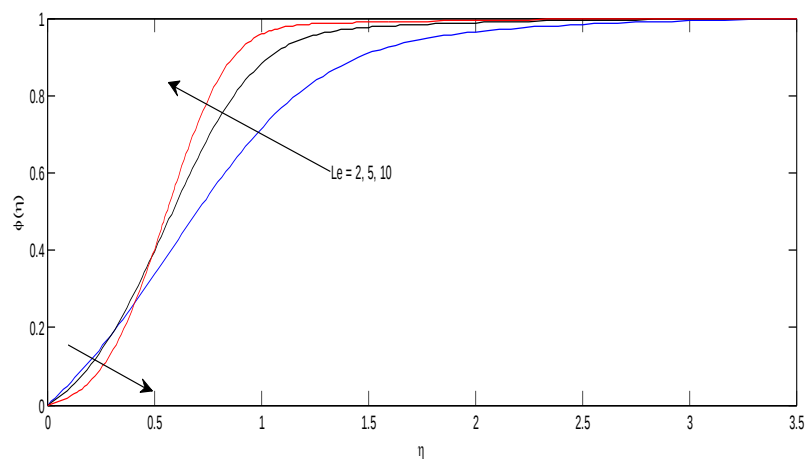
**Figure 18: Concentration profiles for  $Nt$**



**Figure 19: Concentration profiles for  $B$**



**Figure 20: Concentration profiles for  $Kr$**



**Fig. 21 Concentration profiles for various value of  $Le$ .**

Table 1 represents the effects of various parameters on the coefficients of Skin friction, Nusselt number and Sherwood number. It is clearly seen from the table that the coefficients of Skin friction, Nusselt number and Sherwood number are increasing functions of  $M$  and  $Pr$  while these are decreasing function of  $B$ . With the increase in  $Nb$ ,  $Nt$  and  $Le$  the coefficient of Skin friction is getting decreased and the Nusselt number is getting increased. The trend of coefficient of Skin friction and Nusselt number is reversely visible in the case of  $Kr$ . On increasing  $Nt$  the Sherwood number is getting increased while the Sherwood number is getting decreased with the increase in  $Nb$ ,  $Kr$  and  $Le$ . On increasing  $Q$ , the numerical values of skin friction as well as Sherwood number is getting decreased while the values of Nusselt number is getting increased with the increase in  $Q$ . Since  $Pr$  represents the strength of thermal diffusion and as we know, on increasing  $Pr$  the strength of thermal diffusion is getting decreased and vice-versa. This implies that the magnetic field is the cause for the enhancement in coefficient of Skin friction, rate of heat transfer and rate of mass transfer, while the thermal diffusion and melting of the sheet have the reverse effect on these. The Brownian diffusion, Thermophoretic diffusion and Lewis number have the tendency to reduce the coefficient of Skin friction while these have reverse effect on rate of heat transfer. The coefficient of Skin friction gets an enhancement by the chemical reaction. On the other hand the chemical reaction has reverse effect on the rate of heat transfer. Brownian diffusion, chemical reaction and Lewis number are responsible for the decrease in rate of mass transfer while thermophoretic diffusion has the reverse effect on it. Heat absorption has the tendency to reduce the coefficient of skin friction and rate of mass transfer whereas it has reverse effect on the rate of mass transfer.

**Table 1 Coefficient of skin-friction, Nusselt number and Sherwood numbers for various values of flow parameters**

| $M$ | $Pr$ | $Nb$ | $Nt$ | $Q$  | $Le$ | $B$ | $Kr$ | $-C_f \sqrt{Re_x}$ | $-\frac{Nu_x}{\sqrt{Re_x}}$ | $-\frac{Sh_x}{\sqrt{Re_x}}$ |
|-----|------|------|------|------|------|-----|------|--------------------|-----------------------------|-----------------------------|
| 5   | 0.71 | 0.05 | 0.05 | 0.02 | 10   | 2   | 0.01 | 2.438119           | 0.421672                    | 0.042931                    |
| 7   |      |      |      |      |      |     |      | 2.749714           | 0.424787                    | 0.04349                     |
| 10  |      |      |      |      |      |     |      | 3.168629           | 0.428365                    | 0.044178                    |
|     | 0.71 |      |      |      |      |     |      | 2.438119           | 0.421672                    | 0.042931                    |
|     | 1    |      |      |      |      |     |      | 2.520611           | 0.499412                    | 0.095601                    |
|     | 1.5  |      |      |      |      |     |      | 2.603242           | 0.611971                    | 0.217806                    |
|     |      | 0.03 |      |      |      |     |      | 2.440433           | 0.419757                    | 0.063451                    |
|     |      | 0.05 |      |      |      |     |      | 2.438119           | 0.421672                    | 0.042931                    |
|     |      | 0.07 |      |      |      |     |      | 2.435818           | 0.423579                    | 0.033855                    |
|     |      |      | 0.03 |      |      |     |      | 2.439992           | 0.420122                    | 0.031358                    |
|     |      |      | 0.05 |      |      |     |      | 2.438119           | 0.421672                    | 0.042931                    |
|     |      |      | 0.07 |      |      |     |      | 2.43624            | 0.423228                    | 0.054384                    |
|     |      |      |      | 0.01 |      |     |      | 3.3259             | 0.5138                      | 0.1339                      |
|     |      |      |      | 0.02 |      |     |      | 3.3207             | 0.5206                      | 0.1262                      |
|     |      |      |      | 0.03 |      |     |      | 3.3154             | 0.5275                      | 0.1189                      |
|     |      |      |      |      | 5    |     |      | 2.438246           | 0.421567                    | 0.153603                    |
|     |      |      |      |      | 7    |     |      | 2.438164           | 0.421635                    | 0.086589                    |
|     |      |      |      |      | 9    |     |      | 2.438129           | 0.421664                    | 0.052834                    |
|     |      |      |      |      |      | 1   |      | 2.615391           | 0.560604                    | 0.196599                    |
|     |      |      |      |      |      | 1.5 |      | 2.514112           | 0.47971                     | 0.075676                    |
|     |      |      |      |      |      | 2   |      | 2.438119           | 0.421672                    | 0.042931                    |
|     |      |      |      |      |      |     | 0.01 | 2.438119           | 0.421672                    | 0.042931                    |

|  |  |  |  |  |  |  |      |          |          |          |
|--|--|--|--|--|--|--|------|----------|----------|----------|
|  |  |  |  |  |  |  | 0.03 | 2.438143 | 0.421652 | 0.042845 |
|  |  |  |  |  |  |  | 0.05 | 2.438167 | 0.421633 | 0.04276  |

## Conclusions

Following important conclusions are drawn from the present study:

- Heat absorption reduces the nanofluid velocity while it induces the nanofluid temperature.
- Chemical reaction induces fluid velocity while it reduces fluid concentration. It reduces the fluid temperature in the boundary layer region.
- Melting of the sheet has the tendency to reduce nanofluid velocity, temperature and concentration.
- Heat absorption has the tendency to reduce the coefficient of skin friction and rate of mass transfer whereas it has reverse effect on the rate of mass transfer.
- The coefficient of Skin friction gets an enhancement by the chemical reaction. On the other hand the chemical reaction has reverse effect on the rate of heat transfer.
- Melting parameter reduces the coefficient of skin friction, rate of heat transfer and rate of mass transfer.

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