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$\widehat{D}$-irresolute functions and its properties

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Abstract

In this paper, we introduce new type of irresolute functions namely $\widehat{D}$-irresolute function . We obtain their characterizations and their basic properties. Mathematics Subject Classification : 54C05

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1.Introduction

Functions and of course irresolute functions give new path towards research. In 1972, S.G.Crossely and S.K.Hildebrand [3] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the year. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various of mathematics and related sciences. Recently, as generalization of closed sets, the notion of $\widehat{D}$-closed sets were introduced and this notion was further studied by G.Suresh et al. In this paper , we will continue the study of related irresolute functions with $\widehat{D}$-open sets. We introduce and characterized the concepts of $\widehat{D}$-irresolute Function.

2.Preliminaries

Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f:(X,\tau) \longrightarrow (Y,\sigma)$ (or simply $f: X \longrightarrow Y$) denotes a function f of a space (X,τ) into a space (Y,σ). Let A be a subset of a space X. The closure, the interior and complement of A are denoted by $cl(A)$, $int(A)$ and A^c respectively.

Definition 2.1 : A subset A of a topological space (X,τ) is called

- **i)** a pre-open set [6] if $A \subset int(cl(A))$ and a pre-closed set if $cl(int(A)) \subset A$,
- **ii)** a semi-open set [2] if $A \subset cl(int(A))$ and a semi-closed set if $int(cl(A)) \subset A$,
- **iii)** a semi-preopen set [9] (β-open [1]) if $A \subset cl(int(cl(A)))$ and a semi-preclosed set ($= \beta$-closed) if $int(cl(int(A))) \subset A$ and

Definition 2.2 Let (X, τ) be a topological space and $A \subset X$

i) an ω-closed set [10] (=$\hat{g}$-closed [11]) if $cl(A) \subset U$ whenever $A \subset U$ and U is semi-open in (X, τ),
ii) a D-closed set [5] if $pcl(A) \subset int(U)$ whenever $A \subset U$ and U is ω -open in (X, τ).

Complements of the above mentioned sets are called their respectively open sets

Definition 2.3 A subset A of (X, τ) is called an $\widehat{D}$-closed set if $spcl(A) \subset U$ whenever $A \subset U$ and U is D- open in (X, τ) .The class of all $\widehat{D}$-closed sets in (X, τ) is denoted by $\widehat{D}c(\tau)$.That is ,$\widehat{D}c(\tau) = \{ A \subset X : A$ is $\widehat{D}$-closed in $(X, \tau)\}$.

Definition 2.4 Let (X, τ) be a topological space and $A \subset X$

1.semi-pre interior of A denoted by spint(A) is the union of all semi-pre open subsets of A

2..semi-pre closure of A denoted by spcl(A) is the intersection of all semi-pre closed subsets of A

Definition 2.5

A map f: $(X, \tau) \longrightarrow (Y, \sigma)$ is called

(i) strongly-continuous [4] if $f^{-1}(V)$is both open and closed in (X, τ) for each subset of V of (Y, σ),
(ii) perfectly-continuous [7] if $f^{-1}(V)$ is both open and closed in (X, τ) for each open set V of (Y, σ),
(iii) strongly M pre-continuous (SMPC) [8] if $f^{-1}(V)$ is open in (X, τ) for every pre- open set V of (Y, σ) and
(iv) irresolute [3] if $f^{-1}(V)$ is semi-open in (X, τ) for each semi-open set V of (Y, σ),

3. $\widehat{D}$-irresolute Functions

Definition 3.1

A map $f: X \to Y$ is called $\widehat{D}$-irresolute if $f^{-1}(F)$ is $\widehat{D}$-closed in X for every $\widehat{D}$closed set F of Y.

Example 3.2

Let $X = \{a, b, c\} = Y$, $\tau = \{ \emptyset, \{ a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Here $\widehat{D}c(\tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$ and $\widehat{D}c(\sigma) = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ .Clearly the identity map $f: X \to Y$ is $\widehat{D}$-irresolute, since every subset of X is $\widehat{D}$-closed in X.

Proposition 3.3

If $f: X \to Y$ is $\widehat{D}$-irresolute, then f is $\widehat{D}$-continuous but not conversely.

Proof:

Since every closed set is $\widehat{D}$-closed. Hence f is $\widehat{D}$-continuous.

Example 3.4

Let $X = Y = \{a, b, c\}, \tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Here $\widehat{D}c(\tau) = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ and $\widehat{D}c(\sigma) = P(X) - \{a, b\}$. Clearly the identity map $f: X \to Y$ is $\widehat{D}$-continuous but not $\widehat{D}$-irresolute, since $\{a\}$ is $\widehat{D}$-closed in Y but $f^{-1}(\{a\}) = \{a\}$ is not $\widehat{D}$-closed in X.

Proposition 3.5

Let $f: X \to Y$ and $g: Y \to Z$ be any two maps. Then

(a) $g \circ f$ is $\widehat{D}$-irresolute if both f and g are $\widehat{D}$-irresolute.
(b) $g \circ f$ is $\widehat{D}$-continuous if g is $\widehat{D}$-continuous and f is $\widehat{D}$-irresolute.

Proof:

(a): Let $f: X \to Y$ and $g: Y \to Z$ be any two maps. Let F be an $\widehat{D}$closed set in Z. Since g is $\widehat{D}$-irresolute, $g^{-1}(F)$ is $\widehat{D}$-closed in Y. Since f is $\widehat{D}$-irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is $\widehat{D}$-closed in X. Thus $g \circ f$ is $\widehat{D}$-irresolute.

(b) : Let F be a closed set in Z. Since g is $\widehat{D}$-continuous, $g^{-1}(F)$ is $\widehat{D}$-closed in Y. Since f is $\widehat{D}$-irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is $\widehat{D}$-closed in X. Thus $g \circ f$ is $\widehat{D}$-continuous.

Proposition 3.6

Let X be a topological space, Y be a $T_{\widehat{D}}$-space and $f: X \to Y$ be a map. Then the following are equivalent:

(i) f is $\widehat{D}$-irresolute,
(ii) f is $\widehat{D}$-continuous.

Proof:

(i)⇒(ii) : Since every closed set is $\widehat{D}$-closed. Hence f is $\widehat{D}$-continuous.

(ii) ⇒ (i): Let F be an $\widehat{D}$-closed set in Y .Since Y is a $T_{\widehat{D}}$-space, F is a closed set in Y and by hypothesis, $f^{-1}(F)$ is $\widehat{D}$-closed in X. Therefore f is $\widehat{D}$-irresolute.

Definition 3.7

A map $f: X \to Y$ is said to be strongly $\widehat{D}$-continuous if the inverse image of every $\widehat{D}$-open set of Y is open in X.

Proposition 3.8

If a map $f: X \to Y$ is strongly $\widehat{D}$-continuous, then f is continuous but not conversely.

Proof:

Since every open set is $\widehat{D}$-open. Then f is continuous.

Example 3.9

Let $X = Y = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{ \emptyset, \{a\}, Y\}$. Define a map $f: X \to Y$ by $f(a) = a, f(b) = c, f(c) = b$.Clearly f is continuous but not strongly $\widehat{D}$-continuous. Since $\{b\}$ is $\widehat{D}$-open in Y but $f^{-1}(\{b\}) = \{c\}$ is not open in X.

Proposition 3.10

Let X be a topological space, Y be a $T_{\widehat{D}}$-space and $f: X \to Y$ be a map. Then the following are equivalent:

(i) f is strongly $\widehat{D}$-continuous,

(ii) f is continuous.

Proof:

(i)⇒(ii): Since every open set is $\widehat{D}$-open. Then f is continuous.

(ii) ⇒ (i): Let V be any $\widehat{D}$-open set in Y. Since Y is a $T_{\widehat{D}}$-space, V is open in Y. By (ii) $f^{-1}(V)$ is open in X. Therefore, f is strongly $\widehat{D}$-continuous.

Proposition 3.11

Every strongly $\widehat{D}$-continuous map is SMPC, but not conversely.

Proof:

Since every pre open set is $\widehat{D}$-open. Hence every strongly $\widehat{D}$-continuous map is SMPC.

Example 3.12

Let $X = Y = \{a, b, c, d\}, \tau = \{ \emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}, X\}$ and

$\sigma = \{ \emptyset, \{c\}, \{d\}, \{a, c\}, c, d\}, \{a, c, d\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = f(c) = d$. Clearly f is SMPC but not strongly $\widehat{D}$-continuous , since $\{d\}$ is $\widehat{D}$-closed in (Y, σ) and $f^{-1}(\{d\}) = \{d\}$ is not closed in (X, τ)

Proposition 3.13

If a map $f: X \to Y$ is strongly continuous, then f is strongly $\widehat{D}$-continuous.

Proof:

Every $\widehat{D}$-open set is a subset itself. Hence f is strongly $\widehat{D}$-continuous.

The following example supports that the converse of Proposition 3.3.21 is not true.

Example 3.14

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, Y\}$ .Let $f: X \to Y$ be a map defined by $f(a) = b, f(b) = a$ and $f(c) = c$. Clearly f is strongly $\widehat{D}$-continuous but not strongly continuous. Since $f^{-1}(\{a, c\}) = \{b, c\}$ is closed but not open in X.

Definition 3.15

A map $f: X \to Y$ is called perfectly $\widehat{D}$-continuous if the inverse image of every $\widehat{D}$-open set in Y is both open and closed in X.

Proposition 3.16

If a map $f: X \to Y$ is perfectly $\widehat{D}$-continuous then f perfectly continuous (resp. Continuous) but not conversely.

Proof :

Let V be an open set in Y. Then V is $\widehat{D}$-open in Y. Since f is perfectly $\widehat{D}$-continuous, $f^{-1}(V)$ is both open and closed in X .Thus f is perfectly continuous and also continuous .

Example 3.17

Let $X = Y = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Clearly the identity map $f: X \to Y$ is perfectly continuous and continuous but not perfectly $\widehat{D}$-continuous, since the set $\{c\}$ is $\widehat{D}$-open in Y but $f^{-1}(\{c\}) = \{c\}$ is not close and open (clopen) in X.

Proposition 3.18

If $f: X \to Y$ is perfectly $\widehat{D}$-continuous then it is strongly $\widehat{D}$-continuous but not conversely.

Proof :

Let V be $\widehat{D}$-open in Y. Since f is perfectly $\widehat{D}$-continuous, $f^{-1}(V)$ is both open and closed in X .Thus f is strongly $\widehat{D}$-continuous .

Example 3.19

Let (X, τ) and (Y, σ) be defined as in Example 3.3.22. then f is strongly $\widehat{D}$-continuous but not perfectly $\widehat{D}$-continuous, since the set $\{a, b\}$ is $\widehat{D}$-open in Y and $f^{-1}(\{a, b\}) = \{a, b\}$ is open but not closed in X.

From the above discussion we have the following implications:

$$\begin{tikzcd}
\text{perfectly continuous} & \text{perfectly } \widehat{D}\text{-continuous} \arrow[l] \arrow[r] \arrow[d] & \text{strongly } \widehat{D}\text{-continuous} \arrow[r] & \text{SMPC} \\
& \text{continuous} & \text{strongly continuous} \arrow[l] \arrow[u] &
\end{tikzcd}$$

Proposition 3.20

Let X be a discrete topological space, Y be any topological space and $f: X \to Y$ be a map. Then the following are equivalence:

(i) f is perfectly $\widehat{D}$-continuous,
(ii) f is strongly $\widehat{D}$-continuous.

Proof:

(i)⇒(ii) : Let V be $\widehat{D}$-open in Y. Since f is perfectly $\widehat{D}$-continuous, $f^{-1}(V)$ is both open and closed in X .Thus f is strongly $\widehat{D}$-continuous .

(ii) ⇒ (i): Let U be any $\widehat{D}$-open set in Y. By hypothesis $f^{-1}(U)$ is open in X. Since X is a discrete space, $f^{-1}(U)$ is also closed in X and so f is perfectly $\widehat{D}$-continuous.

Proposition 3.21

If $f: X \to Y$ and $g: Y \to Z$ are perfectly $\widehat{D}$-continuous, then their composition $g \circ f: X \to Z$ is also perfectly $\widehat{D}$-continuous.

Proof:

Let $f: X \to Y$ and $g: Y \to Z$ be two maps. Let V be an $\widehat{D}$-open set in Z. Since g is perfectly $\widehat{D}$-continuous. $g^{-1}(V)$ is both open and closed in Y. As f is perfectly $\widehat{D}$-continuous, $f^{-1}\left(g^{-1}(F)\right) = (g \circ f)^{-1}(F)$ is both open and closed in X. Thus $g \circ f$ is perfectly $\widehat{D}$-continuous.

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