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Semi Perfect Disconnected Domination Number in Fuzzy Graphs

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Abstract

A subset $D_{spd}(G)$ of V is said to be a semi perfect disconnected dominating set if $D_{spd}(G)$ is perfect, $< D_{spd}(G)>$ is disconnected and $< V - D_{spd}(G)>$ is connected. The fuzzy perfect disconnected domination number $\gamma_{spd}(G)$ is the minimum fuzzy cardinality taken over all minimal semi perfect disconnected dominating sets of G.

Keywords :Fuzzy graphs, Fuzzy domination, disconnected domination number, Perfect disconnected domination.

Subject classification No. 05C72, 05C75

1. Introduction

Kulli V.R. et.al introduced the concept of perfect domination and perfect disconnected domination in graphs [3]. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness[9].A. Somasundram and S.Somasundram discussed domination in Fuzzy graphs[10]. In this paper we discuss the semi perfect disconnected domination number in fuzzy graphs and obtained the relationship with other known parameters of G.

2. Preliminaries

Definition:2.1

Let G=(V,E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D. The domination number of G is the minimum cardinatliy taken over all dominating sets in G and is denoted by $\gamma(G)$.

Definition: 2.2

Let $G=(\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$for all $u \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u ,v\}) = \mu(\{u ,v\})$ for all $u,v \in V_1$, then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $<V_1>$.

Definition:.2.3

The fuzzy subgraph $H=(\sigma_1, \mu_1)$ is said to be a spanning fuzzy subgraph of $G=(\sigma, \mu)$ if $\sigma_1(u)=\sigma(u)$ for all $u \in V_1$ and $\mu_1(u, v) \leq \mu(u, v)$ for all $u,v \in V$. Let G (σ, μ) be a fuzzy graph and σ_1 be any fuzzy subset of V_1, i.e. ,$\sigma_1(u) \leq \sigma (u)$ for all u.

Definition: 2.4

Let $G=(\sigma,\mu)$ be a fuzzy graph on V. Let u, $v \in V$. We say that u dominates v in G if $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$. A subset D of V is called a dominating set in G if for every $v \notin D$, there exists $u \in D$ such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$or γ.

Definition: 2.5

A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset D′ of D is dominating set of G such that

$$\sum_{v_i \in D'} \sigma(v_i) < \sum_{v_i \in D} \sigma(v_i)$$

Definition: 2.6

The order p and size q of a fuzzy graph G=(σ,μ) are defined to be $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(\{u, v\})$.

Definition: 2.7

An edge e={u, v} of a fuzzy graph is called an effective edge if $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v \in V / \mu(\{u ,v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v \in N(u)} \sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u) / u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u) / u \in V(G)\}$.

Definition: 2.8

A vertex u of a fuzzy graph is said to be an isolated vertex if $\mu(\{u ,v\}) < \sigma(u) \wedge \sigma(v)\}$ for all $v \in V - \{u\}$, that is , $N(u) = \phi$, Thus an isolated vertex does not dominate any other vertex in G.

Definition: 2.9

A set D of vertices of a fuzzy graph is said to be independent if $\mu(\{u,v\}) < \sigma(u) \wedge \sigma(v)\}$for all u , $v \in D$.

Definition: 2.10

The complement of a fuzzy graph G, denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma, \overline{\mu})$ where $\overline{\mu}(\{u, v\}) = \sigma(u) \wedge \sigma(v) - \mu(\{u, v\})$.

Definition: 2.11

Letσ: V→ [0, 1] be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(\{u,v\})=\sigma(u) \wedge \sigma(v)$ for all uv ∈E and is denoted by K_σ.

Definition: 2.12

A fuzzy graph G=(σ,μ) is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2 \in V_1$ or $v_1,v_2 \in V_2$. Further, if $\mu(u,v)=\sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1, σ_2} where σ_1 and σ_2 are the restrictions of σ to V_1 and V_2 respectively.

Definition: 2.13

Let G = (σ, μ) be a regular fuzzy graph on G^*= (V, E). If $d_G(v)$ = k for all v∈V, (i.e ,) if each vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or k-regular fuzzy graph. Where G^*= (V, E) is an underlying crisp graph.

Remark: 2.14

G is k-regular graph iff δ=Δ=k.

Definition: 2.15

Let G= (σ, μ) be a fuzzy graph. The total degree of a vertex u∈V is defined by $td_G(u) = d_G(u) + \sigma(u) = \sum_{uv\in E}\mu(uv) + \sigma(u)$. If each vertex of G has the same total degree k then G is said to be a totally regular fuzzy graph of total degree k or k-totally regular fuzzy graph

Definition: 2.16

A set of fuzzy vertex which covers all the fuzzy edges is called a fuzzy vertex cover of G and the minimum cardinality of a fuzzy vertex cover is called a vertex covering number of G and denoted by β(G).

Definition: 2.17

Let G = (σ, μ) be a fuzzy graph on D and D ⊆ E then the fuzzy edge cardinality of D is defined to be $\sum_{e\in D} \mu(e)$.

Definition: 2.18

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident of 'u' and is denoted by dE(u). $\sum_{v\in N(v)} \sigma(v)$ is called the neighbourhood of u and is denoted by dN(u).

Definition: 2.19

The minimum effective degree $\delta_E(G)$ = min{dE(u) | u ∈V(G)} and the maximum effective degree $\Delta_E(G)$ = max {dE(u) | u ∈V(G)}.

3. MAIN RESULTS

Definition 3.1

Let G = (σ, μ) be a fuzzy graph without isolated vertices. A subset $D_{spd}(G)$of V is said to be a semi perfect disconnected dominating set if $D_{spd}(G)$ is perfect, < $D_{spd}(G)$> is disconnected and < V− $D_{spd}(G)$> is connected. The fuzzy perfect disconnected domination number $\gamma_{spd}(G)$ is the minimum fuzzy cardinality taken over all minimal semi perfect disconnected dominating sets of G.

Example 3.2

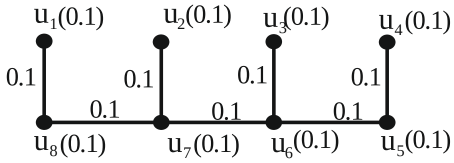

Fig.1

$D_{spd}(G)$ = $\{u_1, u_2, u_3, u_4\}$

<D_{spd}(G))> is disconnected

$<V-D_{spd}(G)>$ is connected

$\gamma_{spd}(G) = 0.4$

Theorem 3.3

If G = (σ, μ) is a fuzzy graph and $\gamma_{spd}(G)$ - set exists, then V– $D_{spd}(G)$ is a perfect dominating set of G.

Proof:

If G = (σ, μ) is a fuzzy graph with vertex set V = { $v_1, v_2, \ldots v_i, v_{i+1}, \ldots, v_n$ }. The semi perfect disconnected dominating set $D_{spd}(G)$, by definition of semi perfect disconnected dominating set every $v_i \in V$ is dominated by exactly one $u_i \in D_{spd}(G)$ and V – $D_{spd}(G)$ contains the equal number of vertices with $D_{spd}(G)$, then by the definition semi perfect dominating set V– $D_{spd}(G)$ is a perfect dominating set of G.

Theorem 3.4

If G = (σ, μ) is a fuzzy graph then $\gamma_{spd}(G)$ + γ□(G) = p where γ□(G) is the inverse domination number of G.

Proof:

If G = (σ, μ) is a fuzzy graph with vertex set V = { $v_1, v_2, \ldots v_i, v_{i+1}, \ldots, v_n$ } and the fuzzy set D(G) and D□(G) are the dominating and inverse dominating sets respectively. Let D_{spd} (G) be the semi perfect disconnected dominating set. By theorem 2.5.3, V– $D_{spd}(G)$ is a perfect dominating set, obviously the dominating set, by definition of inverse dominating set D□(G) $\subseteq$ V – D(G) has a dominating set, then D□(G) is an inverse dominating set of G with respect to D(G). Further, $\gamma_{spd}(G)$ and γ□(G) are the fuzzy the semi perfect disconnected and inverse domination numbers respectively. Therefore $\left|D_{spd}(G)\right| + \left|V - D_{spd}(G)\right| = n$, where n is the number of fuzzy vertices in G, $\gamma_{spd}(G)$ + γ□(G) = p.

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