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Results and Reviews on Domatic Number of Graph

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Abstract

The domatic number of G is the maximum number k such that V (G) can be partitioned into k disjoint dominating set. In this paper, the role of domatic number in various fields is discussed. Applications and Properties of domatic number of a graph are listed. The upper and lower bounds of the domatic number are reviewed. Then some upper bounds of the tree domatic number and the inequalities of upper domatic number are given. Finally, the results on signed domatic number of graphs are showed. The purpose of this paper is study on recent developments in domatic number have been reviewed.

Keywords: Domatic partition, Domatic number, Tree domatic number, Upper domatic number, Signed domatic number.
Mathematical Classifications: *05C05, 05C07, 05C35, 05C69.*

I. Introduction

The domatic number was first formalized by Cockayne and Hedetniemi in 1977 [5]. In 1994, Gerard J. Chang[2] explained the study on domatic numbers of graphs that are obtained from small graphs by performing graph operations. In 2005, Lutz Volkmann, Bohdan Zelinka[4] presented the basic properties of signed domatic number. And also they determined the signed domatic number of complete graphs, cycles, fans, and wheels. In 2007, Xue-gang chen[8] explained some upper bounds for the tree domatic number in term of minimum degree. Then, he established a sharp lower bound for the number of edges in a connected graph of given order. Finally, he shows that a tree domatic number of a planar graph is at most 4 and give a characterization of planar graphs with the tree domatic number 3. In 2008, Peter Dankelmann and Neil Calkin [6] derived upper and lower bounds for the Domatic number. In 2017, the thesis entitled 'A study of the Upper Domatic Number of a Graph' prepared by Nicholas Phillips [5] is an excellent source for research in Upper domatic number. Teresa W. Haynes, Jason T. Hedetniemi, Stephen T. Hedetniemi, Alice McRae, and Nicholas Phillips, are introduced and studied the upper domatic number of a graph. The domatic number problem arises in communication networks. And also arises in various situations of locating facilities and distribute resources in a network.

II. PRELIMINARIES ON DOMATIC NUMBER

In this section, the contributions of several authors towards the field of this fast growing domatic number are presented. In the field of multifacility location and distributed networks are using the domatic number for fixing the station for providing the clear network to their client. In the facility location framework, for instance, there are *k* type of services that all clients in different regions of a city should receive. A graph representing the map of regions in the city is given where the nodes of the graph represent regions and neighboring regions are connected by edges. The facility servers in the city are established using domatic set. Each region can host at most one server such that every client in the city can access that facility server in a particular region or in the neighborhood region. In some cases, it may not be possible to fix a facility location which satisfies the condition that a region in the neighborhood. The required condition is modified to a region at the minimum possible domatic partition set *d*. In particular, Gerard J. Chang gives solutions to the domatic number of the union of two graphs and the domatic number of the join of two or more graphs. He gives partial results of the domatic number of the cartesian product of paths.

Definition 2.1 [6]

In graph theory, a domatic partition of a graph $G = (V, E)$ a partition of V disjoint sets $V_1, V_2, V_3, ..., V_k$ such that each v_i is a dominating set for G.

The maximum number of dominating sets which the vertex set of a graph G can be partitioned is called the *domatic number* of graph G, and it is denoted by $dom\ (G)$ or $d(G)$.

Definition 2.2 [8]

Let G = (V, E) be a simple graph of order n. The *minimum degree* and *maximum degree* of the graph G are denoted by $\delta(G)$ and $\Delta(G)$. Let P_n, C_n and K_n denote the path, cycle and complete graph with n vertices, respectively.

A set of vertices D in a graph G = (V, E) is a *dominating set* if every vertex in V −D has at least one neighbor in D. A dominating set D is called a *tree dominating set* if the sub graph induced by D is a tree. The minimum number of vertices in a tree dominating set of G is called the *tree domination number* of G, and is denoted by $\gamma_{tr}(G)$.

A tree domatic partition of G is a partition of the vertex set V into pair wise disjoint tree dominating set. If such a partition exists, the maximum number of subsets in such a partition is called the *tree domatic number* of G and is denoted by $d_{tr}(G)$.

Definition 2.3 [7]

A vertex partition of V meeting this condition is called a *transitive partition* and a transitive partition of G having order Tr(G) is called a Tr - partition of G.

The *upper domatic number* denoted by D(G), equals the maximum order k of a vertex partition $\pi = \{V_1, V_2, \dots, V_k\}$ such that for all i, j, $1 \le i < j \le k$, either $V_i \to V_j$ or $V_j \to V_i$, or both.

Definition 2.4 [4]

The *signed dominating function* is a two-valued function $f\colon V(G) \to \{-1, 1\}$ such that $\sum_{x\in N[V]} f(x) \ge 1$ for each $v \in V(G)$. The sum $f(V(G))$ is called the weight $w(f)$ of f. The minimum of weights $w(f)$, taken over all signed dominating functions f on G, is called the *signed domination number* of G, denoted by $\gamma_s(G)$.

Let G be a finite and simple graph with the vertex set $V(G)$, and let $f\colon V(G) \to \{-1, 1\}$ be a two valued function. If $\sum_{x\in N[V]} f(x) \ge 1$ for each $v \in V(G)$, where $N[v]$ is the closed neighborhood of v, then f is a signed dominating function on G. A set $\{f_1, f_2, \dots f_d\}$ of signed dominating functions on G with the property that $\sum_{i=1}^{d} f_i(x) \le 1$ for each $x \in V(G)$, is called a signed dominating family (of functions) on G. The maximum number of functions in a signed dominating family on G is the *signed domatic number* on G, denoted by $d_s(G)$.

III. PROPERTIES OF DOMATIC NUMBER OF A GRAPH

Proposition 3.1 [1]

For any graph G, $d(G) \le \delta(G) + 1$.

This result is derived from the basic observation that in a domatic partition, a vertex v, in some set V_i can be dominated by at most deg(v) other sets. From this, we gain the term domatically full used to describe a graph G where $d(G) \le \delta(G) + 1$.

An entire class of graphs that were always domatically full which is found by Chang.

Proposition 3.2 [4]

The signed domatic number $d_s(G)$ is well-defined for each graph G.

Proposition 3.3 [4]

Let G be a graph of order $n(G)$ with signed domination number $\gamma_s(G)$ and signed domatic number $d_s(G)$, then $\gamma_s(G).\, d_s(G) \le n(G)$.

Proposition 3.4 [4]

If G is a graph with minimum degree $\delta(G)$, then $1 \leq d_s(G) \leq \delta(G) + 1$.

Proposition 3.5 [4]

The signed domatic number is an odd integer.

Proposition 3.6 [2]

$d(G_1 \cup G_2) = min\{d(G_1), d(G_2)\}$, For any two graphs G_1 and G_2.

Proposition 3.7 [2]

If x is a dominating vertex of a graph G, then $d(G) = d(G - x) + 1$.

Proposition 3.8 [2]

$d(H) \leq d(G)$, For any spanning subgraph $H = (V, E')$ of $G = (V, E)$

Proposition 3.9 [2]

$d(G\backslash m) \leq d(G\backslash m')$, For any graph G and any nonnegative integers $m \leq m'$.

Proposition 3.10 [7]

For any graph G of order n, 1 ≤ d(G) ≤ Tr (G) ≤ D(G) ≤ n.

Proposition 3.11 [7]

If π = {V_1, V_2, …., V_k} is an upper domatic partition of a graph G, then the partition $\pi' = \{V_1, V_2, \ldots, V_{i-1}, V_i \cup V_j, V_{i+1}, \ldots V_{j-1}, V_{j+1}, \ldots, V_r\}$ obtained from π by deleting V_i and V_j and adding to π' the union $V_i \cup V_j$, for any 1 ≤ i < j ≤ r , is also an upper domatic partition.

Results

1. Let G be a random 3-regular graph. Then $dom(G) \geq 3$. [6]
2. Let G be a random r-regular graph. Then $dom(G) \leq r$. [7]

IV. RESULTS ON UPPER BOUNDS FOR TREE DOMATIC NUMBER

Theorem 4.1 [8]

Let G be a connected graph. Then,

(1) If G is a complete graph, then $d_{tr}(G) = \delta(G) + 1$; otherwise, $d_{tr}(G) \leq \delta(G)$ and the bound is sharp.
(2) If $\gamma_{tr}(G) > 0$, then $d_{tr}(G) \leq \frac{n}{\gamma_{tr}(G)}$ and the bound is sharp.
(3) $d_{tr}(G) \leq K(G)$ and the bound is sharp.

Theorem 4.2 [8]

For any $G(T_1, T_2, T_3, \ldots \ldots, T_k) \in G_t$, there is $d_{tr}(G(T_1, T_2, T_3, \ldots \ldots, T_k)) = k$.

Theorem 4.3 [1]

For any graph G of order n, $d(G)$+ $d(\bar{G}) \leq n + 1$, with equality if and only if $G = K_n$ or $\overline{K_n}$.

Theorem 4.4 [8]

Let G be a connected graph of order n with tree domatic number $k \geq 1$. Then G must have at least $\frac{(k+1)n}{2} - k$ edges. Furthermore, G has exactly $\frac{(k+1)n}{2} - k$ edges if and only if $n \equiv (mod\ k)$ and $G \in G_{\frac{n}{k}}$.

Theorem 4.5 [8]

Let G be a planar graph of order n. Then the tree domatic number of G is at most 4 and K_4 is the only planar graph achieving this bound.

V. REVIEWS ON UPPER DOMATIC NUMBER OF A GRAPH

Results on Inequalities 5.1 [5]

For any graph,

1. $d(G) \leq D(G)$.
2. $T_r(G) \leq D(G)$.
3. $D(G) \leq \psi(G)$.
4. $d(G) \leq T_r(G)$.
5. There is no relation between the upper domatic number of a graph and the achromatic number of a graph.

Theorem 5.2 [7]

For any graph G,

i. $D(G) \geq 3$ if and only if G contains either a K_3 or a P_4.
ii. D(G) = 2 if and only if G is a galaxy with at least one edge
iii. D (G) = 1 if and only if G =$\overline{K_n}$.

Theorem 5.3 [7]

Let G be a graph.

i. Tr (G) = 1 if and only if G =$\overline{K_n}$.
ii. Tr (G) = 2 if and only if G is a galaxy with at least one edge.

Theorem 5.4 [7]

If G is acyclic, then D (G) = Tr (G).

Theorem 5.5 [7]

For any graph G, $D(G) \leq \Delta(G) + 1$.

Theorem 5.6 [7]

There exists a graph G for which D (G) − Tr(G) differ by at least p, for any positive integer p.

VI. SIGNED DOMATIC NUMBER OF GRAPHS

Theorem 6.1 [4]

If $G = K_n$ is the complete graph of order n, then

i. $d_s(G) = n$, if n is odd
ii. $d_s(G) = p$, if n = 2p and p is odd
iii. $d_s(G) = p - 1$, if n = 2p and p is even.

Theorem 6.2 [4]

If G is a wheel of order n, then $d_s(G) = 1$.

Theorem 6.3 [4]

Let C_n be a cycle of length $n \geq 3$. If n is divisible by 3, then $d_s(C_n) = 3$ and $d_s(C_n) = 1$ in the remaining cases.

Theorem 6.4 [4]

Let G be a fan of order n. If n = 3, then $d_s(G) = 3$ and if $n \neq 3$, then $d_s(G) = 1$.

VII. CONCLUSION

In this paper, several results on domatic number have been referred and this will be a compendium for the researchers to work in the field of domatic number.

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